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**STRUCTURAL ANALYSIS OF WELDED TUBULAR TRUSSES IN**  
**FIRE**

Master of Science Thesis

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## ABSTRACT

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The performance structures made of steel under the action of fire is very important in real constructions and there is not an accurate system to estimate the resistance of them at elevated temperatures. Linear elastic analysis is normally used in the design of welded tubular trusses in fire using FEM models. The main question of this study is to define if the elastic linear theory is suitable with statically determined and non-determined trusses.

Statically determined truss is such that support reactions can be calculated just with equilibrium equations only. Statically determined is 'internally' statically non-determined because of the continuous beams for the chords and the eccentricities of the joints. Statically non-determined truss is such that compatibility conditions are needed as well. The strength of the structure and the constraints check are derived from the requirements given in the European building code.

The standard ISO 834 fire is supposed around the tubular truss without fire painting on it. Both linear and non-linear theories are considered in the study using ABAQUS. Non-linear includes both material and geometrical non-linearities. The mechanical analysis is done with constant load and by increasing temperatures given as an input of a previous heat transfer analysis.

The results show that when the truss is externally statically determined the linear model is pretty accurate and can be used to design tubular steel trusses. However, with non-statically determined structures would be needed more in depth studies than the linear elastic analysis.

## PREFACE

This study has been realized at Tampere University of Technology, Department of Structural Engineering, in the Research Centre of Metal Structures. This study has been guided and supervised by the professor Markku Heinisuo.

First of all, I would like to gratefully thank a large number of people who have been helping me in several ways throughout the completion of this study:

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## LIST OF SYMBOLS AND ABBREVIATIONS

$E$	Young's modulus [Pa]
$f_y$	Yield stress [Pa]
$\varepsilon_y$	Yield strain
$I_x, I_y, I_z$	Second moment of area [m <sup>4</sup> ]
$W_{el,y}, W_{el,z}$	Elastic section modulus. y-y and z-z axis [m <sup>3</sup> ]
$W_{pl,y}, W_{pl,z}$	Plastic section modulus. y-y and z-z axis [m <sup>3</sup> ]
$N$	Axial force of member [N]
$Q$	Shear force of member [N]
$M$	Bending moment of member [Nm]
$N_{Rd}$	Design value of the resistance to normal force [N]
$M_{y,Rd}, M_{z,Rd}$	Design values of the resistance to bending moment, y-y and z-z axis [Nm]
$N_{Ed}$	Design normal force [N]
$M_{y,Ed}, M_{z,Ed}$	Design bending moment, y-y and z-z axis [Nm]
$Q_{ed}$	Design shear force [N]
$Q_{pl}$	Plastic shear resistance [N]
$v$	Deflection [m]
$v''$	Curvature
$A$	Area of the cross-section [m <sup>2</sup> ]
$A_{eff}$	Effective area of the cross-section [m <sup>2</sup> ]
$N_{cr}$	Elastic critical force for the relevant buckling curve [N]
$L_{cr}$	Buckling length [m]
$N_{b,Rd}$	Design buckling resistance of a compression member [N]
$\chi$	Reduction factor for the relevant buckling curve
$\Phi$	Value to determine the reduction factor $\chi$
$\alpha_{cr}$	Imperfection factor
$\bar{\lambda}$	Non-dimensional slenderness
$k_{yy}$	Interaction factor
$\gamma_{M1}$	Partial factor for resistance of members to instability assessed members checks
$C_{my}$	Equivalent uniform moment factor
$\rho$	Density [kg/ m <sup>3</sup> ]
$k$	Thermal conductivity [W/mK]
$C_p$	Specific heat [J/kgK]
$\alpha_L$	Coefficient of linear thermal expansion [m/mK]
$\Delta\varepsilon$	Strain variation
$\varepsilon_{th}$	Thermal strain
$\varepsilon_\sigma$	Stress-related strain

$\varepsilon_{cr}$	Creep strain
$\sigma$	Stress for unit load [Pa]
$T$	Members temperature [°C]
$t$	Profile wall thickness [mm]
$b$	Profile width [mm]
$f_{y,\theta}$	Reduced Effective yield strength [Pa]
$f_{p,\theta}$	Reduced Proportional limit [Pa]
$E_{a,\theta}$	Reduced Slope of the linear elastic range [Pa]
$\theta$	Reduced by temperature
$\varepsilon_{p,\theta}$	Strain at the proportional limit
$\varepsilon_{y,\theta}$	Yield strain
$\varepsilon_{t,\theta}$	Limiting strain for yield strength
$\varepsilon_{u,\theta}$	Ultimate strain
$k_{y,\theta}$	Reduction of yield strength of the steel
$k_{E,\theta}$	Elastic limit reduction
$k_{p,\theta}$	Proportional limit reduction
$\Theta_a$	Steel temperature [°C]
$L$	Member length at 20 °C [m]
$\Delta L$	Member length variation with temperature [m]
$h_i$	Profile height [mm]
$L$	Total length [m]
$H$	Total height [mm]
$g$	Gap dimension [mm]
$\alpha$	Inclination [1/m]
$h_{uc}, h_{bc}$	Chords height profile [mm]
$e_x, e_y$	Eccentricity of the joint [mm]
$b_i$	Overall out-of-plane width of RHS member [mm]
$b_{ep}$	Effective width of an overlapping brace to overlapped brace connection [mm]
$b_{eff}$	Effective width for a brace member to chord connection [mm]
$A_v$	Shear area [m <sup>2</sup> ]
$V_{ed}$	Design shear force [N]
$V_{pl}$	Plastic shear resistance [N]
$N_{i,Ed}$	Design action checked at joints [N]
$N_{i,Rd}$	Resistances values checked at joints [N]
$N_{0,gap,Rd}$	Chord shear resistance value [N]
$N_{0,Ed}$	Design shear action on the chord [N]
$\Phi$	Most unfavorable angle
$K$	Stiffness of the spring [N/m]

$V$	Volume of the member per unit length [ $\text{m}^3$ ]
$\rho$	Density of the material [ $\text{kg}/\text{m}^3$ ]
$c$	Specific heat of the member [ $\text{Ws}/\text{kgK}$ ]
$\Delta T_s$	Temperature change of the member during the time step [ $^{\circ}\text{C}$ ]
$\varepsilon$	Emissivity of the surface of the member
$\sigma$	Stefan-Boltzmann constant [ $\text{W}/\text{m}^2\text{K}^4$ ]
$h$	Convective heat transfer [ $\text{W}/\text{mK}$ ]
$A_m$	Surface area per unit length of the cross-section [ $\text{m}^2$ ]
$T_g$	Gas temperature [ $^{\circ}\text{C}$ ]
$\eta_{fi}$	Finnish reduction factor
$E_d$	Design action value
$E_{d,fi}$	Design action value in Finland
$\psi_{1,1}, \psi_{2,1}$	Combination factors
$Q_{k,1}$	Characteristic value of the leading variable action
$G_k$	Characteristic value of a permanent action
$\gamma_G$	Partial factor for permanent actions
$\gamma_{Q,1}$	Partial factor for variable action 1.
MPC	Multi point constrains
FEM	Finite element method
SHS	Square hollow section
NLMAT	Non-linearity on material
NLGEOM	Non-linearity in geometry



# 1. INTRODUCTION

## 1.1 Background

Steel is, nowadays, one of the most used materials for construction in the world and it is the one which can make possible reach new limits on buildings and other structures. It is a material with a lot of advantages and very few disadvantages. The main features of using steel in construction are the lightness of the structures and the rapidity of mount in comparison with other materials. It is also a very sustainable material environmentally talking.

Nevertheless it has two crucial disadvantages: It needs protections and care with corrosion and protection against fire as well. Otherwise the steel structure will lose all its properties faster and would not be able to stand the service life.

The fire situation is one of the keys of this thesis and besides, one of the most important challenges in the study and construction of any steel structure. When fire is acting and the temperature of members grows they become weaker and more flexible, two non-desired adjectives for any structure (Choi, Burgess, & Plank, 2008).

However, an unprotected steel structure could have fire resistance enough and perform well in some cases since the whole steel structure has the ability of redistribute loads through the structure itself. To assess and understand the real performance of the steel structure is needed to really understand the behavior of the supports conditions, which are representing the action of various other elements.

In order to understand how the steel structure works itself, and besides all the ongoing researches for steel design and its thermal properties, it has become fundamental the use of computer programs such as ABAQUS with which make possible to model and study the structure using FEM.

## 1.2 Aim

The purpose of this thesis is to study the behavior of a Warren-type welded tubular truss, typical in Finnish constructions, with the aid of finite elements software (Yang, Lin, Leu, & Huang, 2008).

The main purpose is to study the validity of linear studies, and compare them with non-linear analysis of the same structure taking account on the different support condi-

tions and realizing on the importance of the situation of the whole structure and the different behavior of it while being externally statically determined or not.

### 1.3 Objectives

The first objective of the study is the design and check of feasibility of the structure in ambient conditions and under fire situation. The fire situation of the welded tubular roof is modeled with ISO 835 fire progressing up to the convergence stop. The resistance of both members and joints is checked using the rules of Eurocodes (EN 1993-1-1, 2005), (EN 1993-1-2, 2005) and (EN 1993-1-8, 2005). In the mechanical analysis both linear and non-linear theory is used, where non-linearity means material and geometrical non-linear behavior. The idea is to know how accurate are linear analysis on those kind of structures. During the analysis intumescent paint is not used for the fire protection. The simulation is carried out at both ambient and elevated temperatures by employing ABAQUS. The joints of trusses to the surrounding structures, and the support conditions of the surrounding structures, are extremely important as shown in many aforementioned studies, such as (Quintiere, di Marzo, & Becker, 2002), (Usmani, Chung, & Torero, 2003), (Choi, Burgess, & Plank, 2008) and (Pada, 2012).

A second objective is to investigate the reliability of statically determined studies, comparing them with not statically determined and check if it is necessary the realization of them. All the loads cases will be performance in order to study the different responses of the truss with the dissimilar support conditions.

The support conditions studied are simply supported and axially non-restrained, simply supported and axially restrained (Liu, Zhao, & Jin, 2010) and fully restrained supports. Buckling problem in the truss member will be checked since is often the first mode of the collapse (Flint, Usmani, Lamont, Torero, & Lane, 2006).

## 2. THEORETICAL BACKGROUND

### 2.1 Structural analysis. Global analysis methods

Non-linear problems are very interesting on engineers work, as well as for physicist, mathematicians and many other scientist since non-linear systems are inherently in nature. Non-linear problems are pretty difficult to solve in comparison with linear, but there are some phenomena such as chaos and singularities that are hidden if non-linearity is not taking into account (Kellert, 1993).

The structural analysis consist on achieving in the whole structure the effect of the actions in order to guarantee the service and ultimate limit states defined in each situation.

The use of appropriate structural models that take into account all relevant variables (such as safety factors, geometry of the structure, materials used, bending...) is needed to perform that analysis. It will provide results at sectional and global level, including reactions, displacements and mises.

It works for, furthermore, to estimate the local behavior (stress-strain) of those areas where the classic hypothesis of material resistance is not applicable as upcoming local areas to concentrate loads, sections changes.

Model and fundamental hypothesis should be adopted in the structural analysis in order to approximate to the real behavior of structures with the accuracy needed to ensure the considered limit state verification.

#### 2.1.1 Global analysis method

Every structural analysis should satisfy compatibility and equilibrium conditions, considering the material's laws of behavior. The methods accepted to calculate and analyze a structure in the global analysis can be classified as follows:

##### 2.1.1.1 Linear analysis

Linear analysis is based on the elastic-linear material's behavior hypothesis and considering the equilibrium in the non-deformed shape of the structure (first order analysis consideration).

These analyzes are the most used in steel structures analysis because of its simplicity. They consider a linear response of the structure where superposition of the effects of different actions and total reversibility of deformation are accepted.

Since the results in the static section are not very sensitive to small variations the exact dimensioning of the structure is not necessarily needed.

### **2.1.1.2 Non-linear analysis**

Non-linear analysis consists in an analysis where non-linear behavior of materials and geometric non-linearity are considered. That is the consideration of the equilibrium in the deformed shape of the structure (second order analysis). In these non-linear analyses could be considered one or both of the before mentioned causes of non-linearity (material or geometry).

The principle of superposition is invalid in this kind of analysis in those cases where the structural response depends on the load, covering elastic and elastoplastic behavior until the exhaustion of the structure.

In non-linear analysis, for a given load, an iterative process of successive linear analysis is done in order to converge on a solution that satisfies the conditions of equilibrium, compatibility and material behavior. These conditions are checked in some of the structure sections (depends on discretization of the analysis) that should be accurate enough to ensure an adequate approximation of the structural response.

### **2.1.2 Material non-linearity consideration**

Material non-linearity effects can be classified depending on the type of analysis. They could be:

- Elastic analysis
- Plastic analysis
- Elastoplastic analysis

In the linear analysis, elastic behavior of materials is going to be considered. It is founded on the supposition that, under a load, the deformation of the material will be linear, so the deformations are proportional to the stress, as can be seen in Figure 2.1(a constant value of elastic modulus is used). The superposition principle is, therefore, accepted in this analysis.

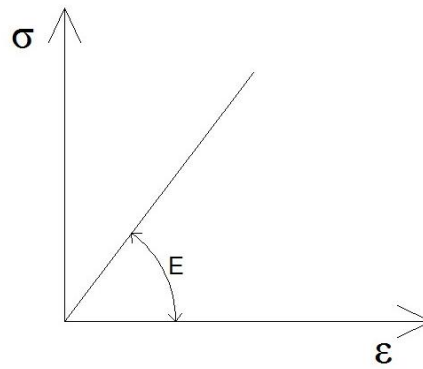


Figure 2.1.-Linear elastic behavior.

When plasticity is considered, it is assumed that the plastic stress is distributed around the section (plastic hinges formation) and also a redistribution of the bending moment needed to create and allow the development of all the hinges required to make the plastic mechanism.

In this research, when non-linearity of the materials behavior is considered, bi-linear behavior is actually considered. The cross section will remain elastic deformation until the yield is reached. The section will continue being deformed with the same load (constant) at the plastic range.

Since in this research fire situation is considered as well, the steel properties will change with the temperature, the stress-strain diagram and the Young's modulus. So when non-linearity of material behavior is considered, a diagram like the one shown below (Figure 2.2) is going to be used in the analysis. The strain-hardening and necking are not considered.

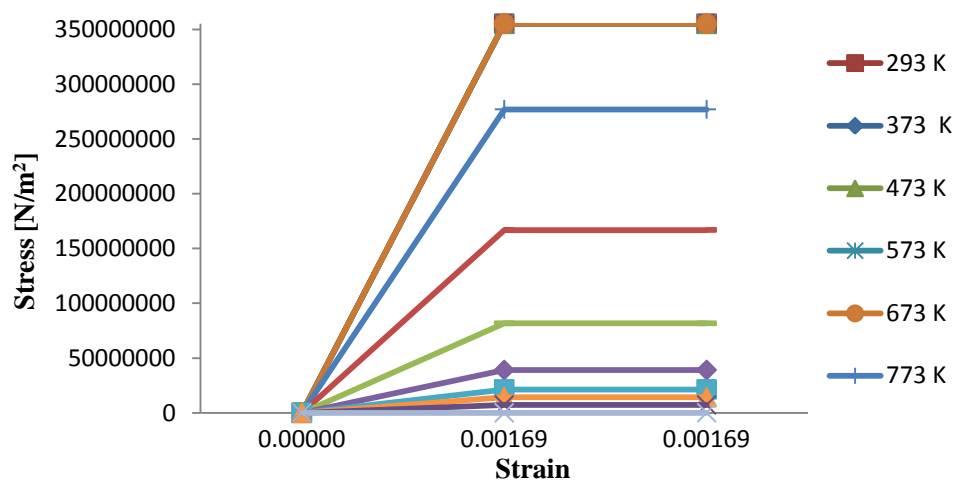


Figure 2.2.-Plastic behavior of steel at elevated temperature (EN 1993-1-2, 2005).

## 2.2 Buckling resistance of members

Buckling is a mathematical instability of members, leading to a failure mode. In some situations, under an increasing load, could be founded two equilibrium states: an undeformed state or a laterally-deformed situation.

In real cases, buckling is considered as a sudden failure of a structural member of a structure subjected to high compressive stress (as shown in Figure 2.3). In those cases the compressive stress in that failure point is less than the ultimate compressive stress that the material can withstand.

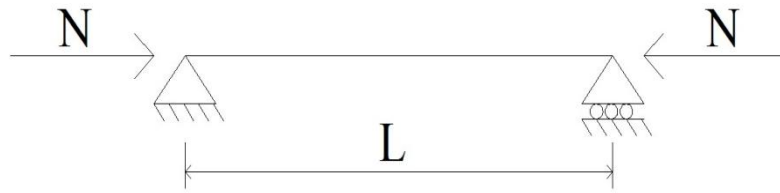


Figure 2.3. - Simply supported beam with compressive axial force.

Considering little deformations and the equilibrium in the deformed shape of the structure (second-order analysis), the next equation is used in the Eurocode (EN 1993-1-1, 2005):

$$E \cdot I_x \cdot v''(x) - N \cdot v(x) = 0 \quad (2.1)$$

where  $v$  is the deflection ( $x$  is the coordinate along the member axis).

$$v = A \cdot \sin kx \quad (2.2)$$

$$k = \sqrt{\frac{N}{E \cdot I_x}} \quad (2.3)$$

$A$  is a constant,  $N$  is the axial force of member,  $E$  is the Young's modulus and  $I_x$  is the second moment of area.

Then we have the typical eigenvalue problem when applying the boundary conditions of the model. The resultant equation achieved for the critical load is given below (EN 1993-1-1, 2005):

$$N_{cr} = \frac{\pi^2 EI}{L^2} \quad (2.4)$$

This is just the upper limit for the axial force that can be resisted by the members of a structure. The assumption of ideal situation is considered in this previous equation, but it is not totally considered in these analyses.

Since a member is conditioned by a lot of imperfections, the ultimate load is calculated by taking into account these imperfections. The imperfections of members are due to:

- The axis of the member will never be perfectly straight. There will be an inevitably displacements on the initial geometry;
- The load is never totally centered on the structure. Some eccentricity will be applied with the loads inevitably;
- The material behavior is not going to be perfectly linear and elastic;
- The manufacturing processes of members and the conditions of the environment induce the member to some stress that affect the real behavior of it.

## 2.2.1 Uniform members in compression

### 2.2.1.1 Buckling resistance

Following the Eurocode, a compression member of a structure should be verified with the equation below (EN 1993-1-1, 2005):

$$\frac{N_{Ed}}{N_{b,Rd}} \leq 1.0 \quad (2.5)$$

where  $N_{Ed}$  is the design value for the compression force in the member and  $N_{b,Rd}$  is the buckling resistance of the same member in compression.

The buckling resistance of each member must be calculated with the next equations (EN 1993-1-1, 2005).

$$N_{b,Rd} = \frac{\chi A f_y}{\gamma_{M1}} \quad \text{for Class 1, 2 and 3 cross-sections.} \quad (2.6)$$

$$N_{b,Rd} = \frac{\chi A_{eff} f_y}{\gamma_{M1}} \quad \text{for Class 4 cross-sections.} \quad (2.7)$$

where  $\chi$  is the reduction factor depending on the mode of buckling,  $f_y$  is the yield stress,  $\gamma_{M1}$  is the partial factor for resistance of members to instability assessed members checked,  $A$  is the Area of the cross-section and  $A_{eff}$  is the effective area of the cross-section.

### 2.2.1.2 Buckling curves

The values needed for the reduction factor  $\chi$  for an appropriate non-dimensional slenderness  $\bar{\lambda}$  should be calculated from Equation 2.8. See also Figure 2.4:

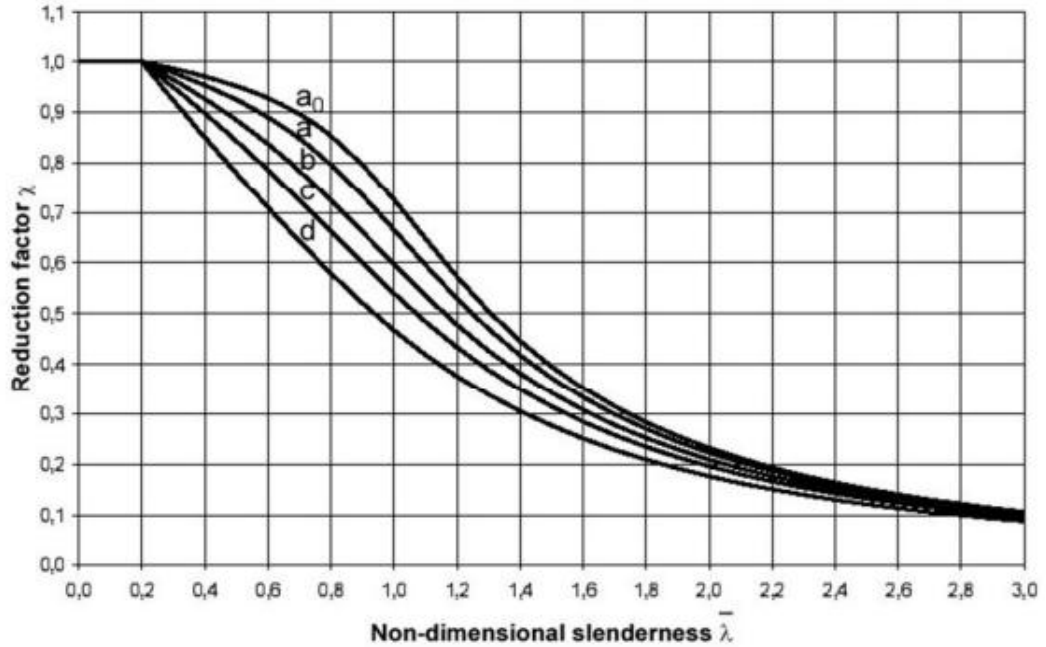


Figure 2.4.- Buckling curves (EN 1993-1-1, 2005).

Before that, the values for the reduction factor  $\chi$  should be obtained using the next equation also given in the Eurocode (EN 1993-1-1, 2005).

$$\chi = \frac{1}{\Phi + \sqrt{\Phi^2 - \lambda^2}} \quad (2.8)$$

where

$$\Phi = 0,5[1 + \alpha(\lambda - 0,2) + \lambda^2] \quad (2.9)$$

$$\lambda = \sqrt{\frac{Af_y}{N_{cr}}} \quad \text{for Class 1, 2 and 3 cross-sections} \quad (2.10)$$

$$\lambda = \sqrt{\frac{A_{eff}f_y}{N_{cr}}} \quad \text{for Class 4 cross-sections} \quad (2.11)$$

where  $\alpha$  is the imperfection factor and  $N_{cr}$  is the critical elastic force based on the cross sectional properties.



The imperfection factor  $\alpha$  depends on the appropriate buckling curve chosen and can be obtained from the Table 2.1 and the selection of the buckling curve used is done using Figure 2.5

Table 2.1. - Imperfection factors for buckling curves (EN 1993-1-1, 2005).

Buckling curve	$a_0$	a	b	c	d
Imperfection factor $\alpha$	0.13	0.21	0.34	0.49	0.76

For slenderness  $\bar{\lambda} \leq 0.2$  or for  $\frac{N_{Ed}}{N_{cr}} \leq 0.04$  buckling effect should not be considered and only the cross sectional check is applied.

Cross section		Limits	Buckling about axis	Buckling curve		
				S 235 S 275 S 355 S 420	S 460	
Rolled sections		$h/b > 1,2$	$t_f \leq 40\text{mm}$	y-y z-z	a b	$a_0$ $a_0$
			$40\text{mm} < t_f \leq 100$	y-y z-z	b c	a a
		$h/b \leq 1,2$	$t_f \leq 100$	y-y z-z	b c	a a
			$t_f > 100$	y-y z-z	d d	c c
Welded I-sections		$t_f \leq 40\text{mm}$	y-y z-z	b c	b c	
		$t_f > 40\text{mm}$	y-y z-z	c d	c d	
Hollow sections		hot finished	any	a	$a_0$	
		cold formed	any	c	c	
Welded box sections		generally (except as below)	any	b	b	
		thick welds: $a > 0,5t_f$ $b/t_f < 30$ $h/t_w < 30$	any	c	c	
U-, T- and solid sections			any	c	c	
L-sections			any	b	b	

Figure 2.5. - Selection of buckling curve for a cross-section (EN 1993-1-1, 2005).

## 2.2.2 Uniform members in bending moment and compression

### 2.2.2.1 Buckling resistance

A member which is laterally unrestrained and subjected to major axis bending moment and compression must be verified following Eurocode (EN 1993-1-1, 2005) while using the next verification equation:

$$\frac{N_{Ed}}{\frac{\chi_y A f_y}{\gamma_{M1}}} + \frac{k_{yy} M_{y,Ed}}{\frac{W_{pl,y} f_y}{\gamma_{M1}}} \leq 1 \quad (2.12)$$

where

- $\chi_y$  is the reduction factor due to the compressive force;
- $k_{yy}$  is the interaction factor;
- $W_{pl,y}$  is the plastic modulus of the section.

The values for those interaction factors are calculated as shown below (EN 1993-1-1, 2005) and using Figure 2.6:

$$k_{yy} = C_{my} \cdot \min \left\{ 1 + (\bar{\lambda}_y - 0.2) \frac{N_{Ed}}{\chi_y N_{pl}}; 1 + 0.8 \frac{N_{Ed}}{\chi_y N_{pl}} \right\} \quad (2.13)$$

$$C_{my} = \begin{cases} \max \left\{ 0.1 + 0.8 \frac{M_{ed,min}}{M_{ed,max}}; 0.4 \right\} & \text{if compression} \\ 1 & \text{if tension} \end{cases} \quad (2.14)$$

Interaction factors	Type of sections	Design assumptions	
		elastic cross-sectional properties class 3, class 4	plastic cross-sectional properties class 1, class 2
$k_{yy}$	I-sections RHS-sections	$C_{my} \left( 1 + 0,6 \bar{\lambda}_y \frac{N_{Ed}}{\chi_y N_{Rk} / \gamma_{M1}} \right)$ $\leq C_{my} \left( 1 + 0,6 \frac{N_{Ed}}{\chi_y N_{Rk} / \gamma_{M1}} \right)$	$C_{my} \left( 1 + (\bar{\lambda}_y - 0,2) \frac{N_{Ed}}{\chi_y N_{Rk} / \gamma_{M1}} \right)$ $\leq C_{my} \left( 1 + 0,8 \frac{N_{Ed}}{\chi_y N_{Rk} / \gamma_{M1}} \right)$
$k_{yz}$	I-sections RHS-sections	$k_{zz}$	$0,6 k_{zz}$
$k_{zy}$	I-sections RHS-sections	$0,8 k_{yy}$	$0,6 k_{yy}$
$k_{zz}$	I-sections	$C_{mz} \left( 1 + 0,6 \bar{\lambda}_z \frac{N_{Ed}}{\chi_z N_{Rk} / \gamma_{M1}} \right)$ $\leq C_{mz} \left( 1 + 0,6 \frac{N_{Ed}}{\chi_z N_{Rk} / \gamma_{M1}} \right)$	$C_{mz} \left( 1 + (2\bar{\lambda}_z - 0,6) \frac{N_{Ed}}{\chi_z N_{Rk} / \gamma_{M1}} \right)$ $\leq C_{mz} \left( 1 + 1,4 \frac{N_{Ed}}{\chi_z N_{Rk} / \gamma_{M1}} \right)$
	RHS-sections		$C_{mz} \left( 1 + (\bar{\lambda}_z - 0,2) \frac{N_{Ed}}{\chi_z N_{Rk} / \gamma_{M1}} \right)$ $\leq C_{mz} \left( 1 + 0,8 \frac{N_{Ed}}{\chi_z N_{Rk} / \gamma_{M1}} \right)$
For I- and H-sections and rectangular hollow sections under axial compression and uniaxial bending $M_{y,Ed}$ the coefficient $k_{zy}$ may be $k_{zy} = 0$ .			

Figure 2.6.-Interaction factors  $k_{ij}$  for members (EN 1993-1-1, 2005).

## 2.3 Steel trusses

In steel structures it is very common the use of trusses, not only in light structures, but also in very heavy and big structures such as bridges. The main advantage while using trusses is the small amount of material used in comparison with other structures. The truss is also a good option since it transfers both tensile and compressive forces, it needs less material and it means smaller cost and lower self-weight. An example of a steel truss is shown in Figure 2.7.

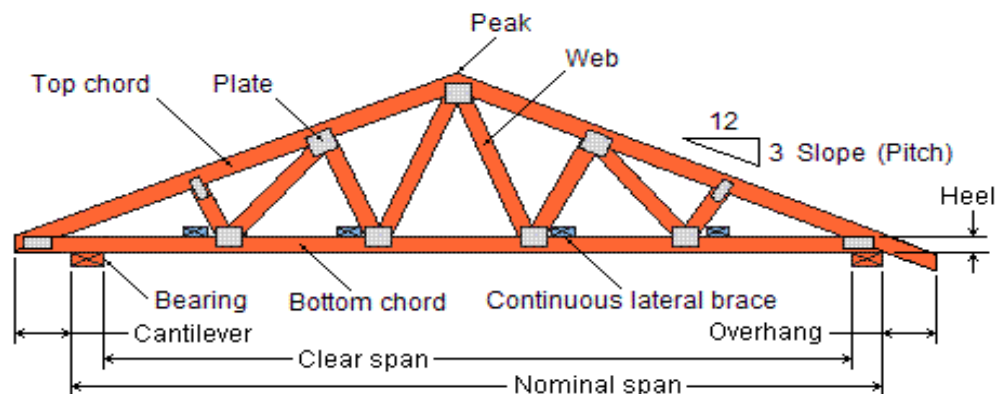


Figure 2.7. - Steel truss example, details (Alpine Engineered Products, 2000).

When a truss is analyzed, normally, the loads are applied to the joints only and not at intermediate points along the members of it. Comparing the loads with the members' weight, this weight is normally omitted because it is insignificant. In some cases, half of the weight of each member is applied as an external load on its two end joints.

Since the members are long and slender, the moments transmitted are considered as zero in the joints and they can be considered as hinges or "pin-joints" (Martini, 2010). As a result, there is no bending moment in the members of those kinds of structures. This makes trusses also quite easy to analyze and calculate the actions. Consequently it makes also the trusses physically stronger than other structures as almost all the construction materials can hold much larger loads in tension and compression than in bending, torsion and shear.

The structural analysis of a truss can be done in three different ways: The direct stiffness method, the flexibility method or with the finite element method. In this study the finite element method is used. The chords are modeled as continuous members, as shown later.

### 2.3.1 Different types of trusses

There are a large number of different truss structures that can be used depending on the type of action, situation or load (Chinnis, 2013). The maximum span of the truss depends also on the truss type.

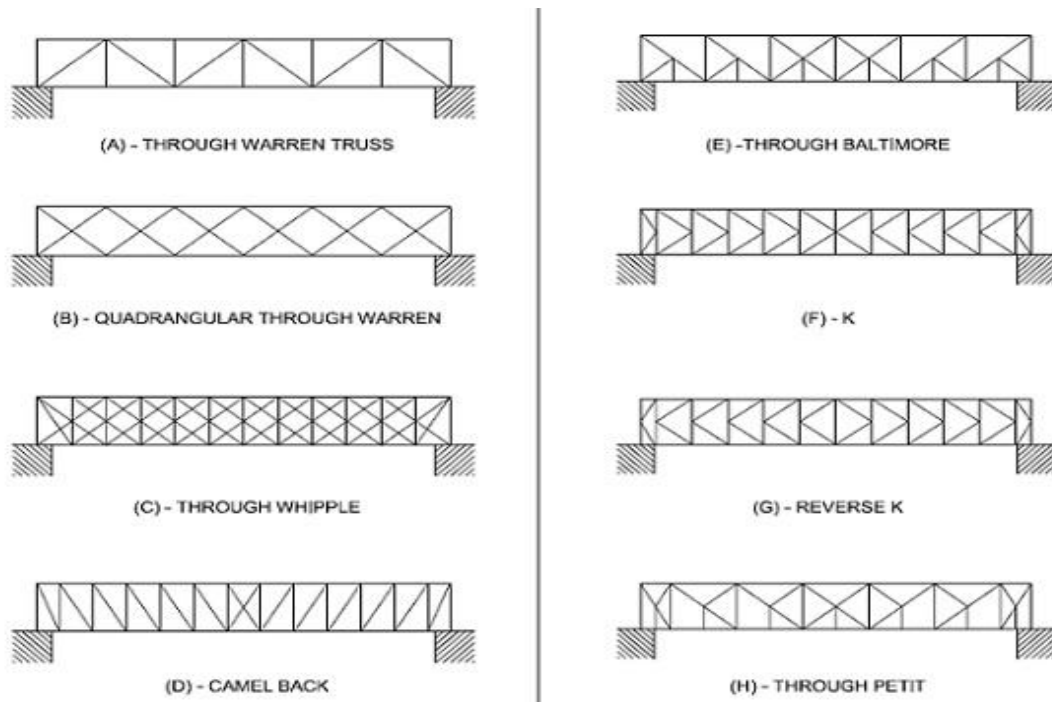


Figure 2.8.-Different standard truss configurations (Chinnis, 2013).

A great variety of type structures is possible depending on the diagonal type of the structure. This style should be chosen depending on how the truss is loaded, if the load is uniform and continuous along the members and how the truss is connected to the supports (columns). In Figure 2.8 can be seen the different standard configurations that are usually used.

### 2.3.2 Truss elements and joints

The designer of the truss is the attendant to choose the cross sectional shape of the chords and diagonals. This choice should be made taking into account the direction and character of the load and what kind of joint is the truss made with, bolted or welded.

How to fix the connection between the chords and the diagonals is another important part of the design of the truss. The connection could be done with bolts or with welds and the braces could either be directly fastened to the chords or to steel plates.

In the analysis, the truss can be thought as a big beam. In the truss, the lower and the upper chords carry tension and compression (depending on the direction of the bending of the whole structure).

The important feature in a truss is that the diagonals carry only axial forces, while in the chords shear and bending moment is also present. Depending on the direction of the forces some elements would be in tension and other in compression. The design should take this into consideration carefully since in compressive members some buckling problems are going to be present. In trusses with dynamic loads this is even more important.

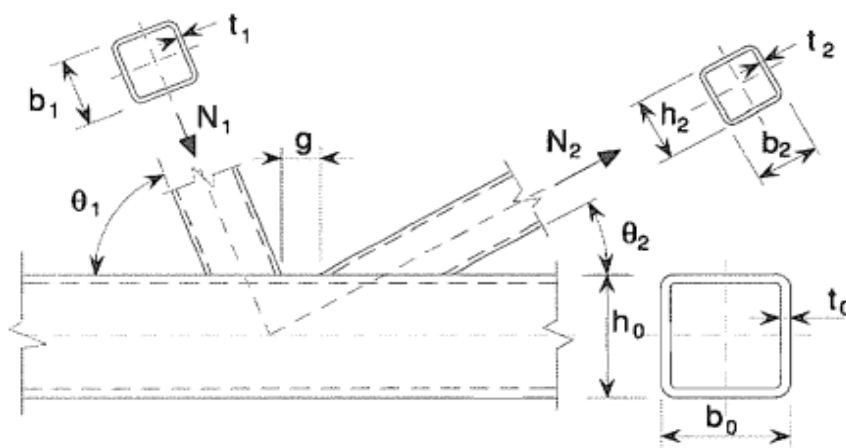


Figure 2.9. - K-joint between RHS braces and RHS chord (EN 1993-1-8, 2005).

After deciding the type of the truss the minimum cross sections needed in the members should be defined, the last step is to design the joints. Joints can be considered as rigid, semirigid or hinged. In this study the braces are supposed to be connected to the chords with hinges, as given in EN 1993-1-8. The chords are modeled as continuous beams. In the designing of joints is also important the eccentricity of the hinge (Boel, 2010). In this study the eccentricities at the joints are taken into account as is required in EN 1993-1-8. In Figure 2.9 can be seen an example of a K-joint as the studied in this research.

### 3. STEEL PROPERTIES AT ELEVATED TEMPERATURES

#### 3.1 Introduction

Steel is a metal whose most important element is iron, and carbon is its primary alloying component. In this section an overview of the properties of steel is provided, both mechanical and thermal.

When the steel starts to yield the stress-strain curve is no longer in the elastic range. A typical image of the structural behavior of steel is shown in Figure 3.1. Different steel grades have different yield point. Sometimes, if the elastic limit or the ultimate yield point is not well defined, the yield point is considered where the elongation of the steel is 0.2%.

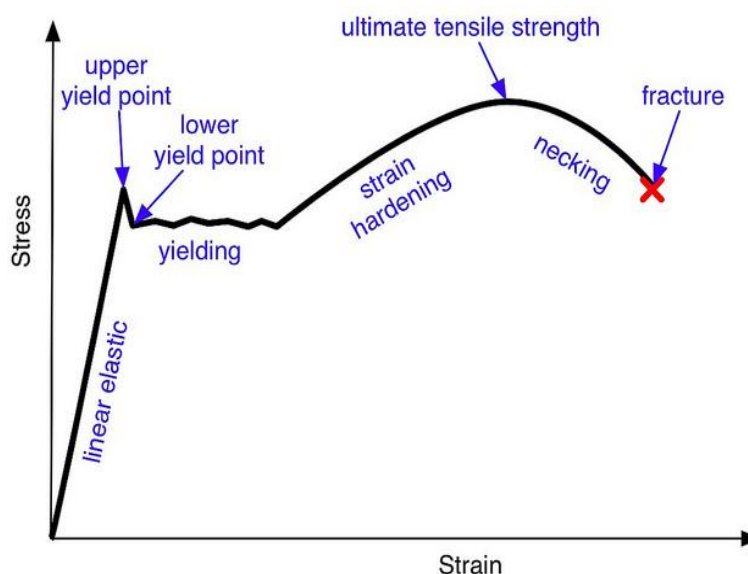


Figure 3.1. - Stress-strain curve of steel.

Steel is a very good conductor of heat. Conduction, which is one of the most important ways of heat transfer, is because the percentage of iron in the steel, which has free electrons that propagate heat through elements (such as beams, columns, panels...).

Steel properties are very different in elevated temperature. These properties are going to be exposed to make easier to understand and develop the models studied.

## 3.2 Definitions

### 3.2.1 Density

Density is considered as a physical property of material. The definition of density is the heaviness of an object or thing while the volume is constant. The formula of density is the mass divided by volume. Density is a temperature and pressure dependent property. Increasing one of them make a change in the volume of the object, and the in density. Density is normally denoted as  $\rho$ . Units of density are  $\text{kg/m}^3$ .

### 3.2.2 Thermal Conductivity

The property of materials to conduct heat is called thermal conductivity. The definition of it is the amount of heat flux that can pass through a material. It depends on the temperature gradient. The reciprocal property of a material is called thermal resistivity. It is evaluated at the Fourier's Law. Thermal conductivity is normally denoted as  $k$  or  $\lambda$ . Units of conductivity are  $\text{W/mK}$ .

### 3.2.3 Specific Heat

The heat needed to raise the temperature of one gram of a material by one degree Celsius is the definition of Specific Heat. The usual denotation is  $C_p$  and its normal units are  $\text{J/kgK}$ .

### 3.2.4 Thermal expansion

Expansion is the tendency of one material to change its volume while the temperature is changing. It makes members to get increases in volume and length. While matter is heated, its particles start moving more and faster and it makes that rise of volume.

The degree of expansion divided by the increase of temperature is the formula of the coefficient of thermal expansion and it normally depends on the temperature. It is denoted as  $\alpha_L$  and unit used are  $\text{m/mK}$ .

## 3.3 Steel mechanical properties

### 3.3.1 Components of the strain

Strain is the elongation of an object compared to its original length. It is dependent with the stress on the object and temperature. The variation in strain with temperature is defined as follows:

$$\Delta\varepsilon = \varepsilon_{th}(T) + \varepsilon_{\sigma}(\sigma, T) + \varepsilon_{cr}(\sigma, T, t) \quad (3.1)$$



where

- $\Delta\varepsilon$  is the change in strain;
- $\varepsilon_{th}$  is the thermal strain;
- $\varepsilon_{\sigma}$  is the stress-related strain;
- $\varepsilon_{cr}$  is the creep strain.

This variation is shown in the next picture, where the stress-strain curve is changing with elevated temperatures (Figure 3.2).

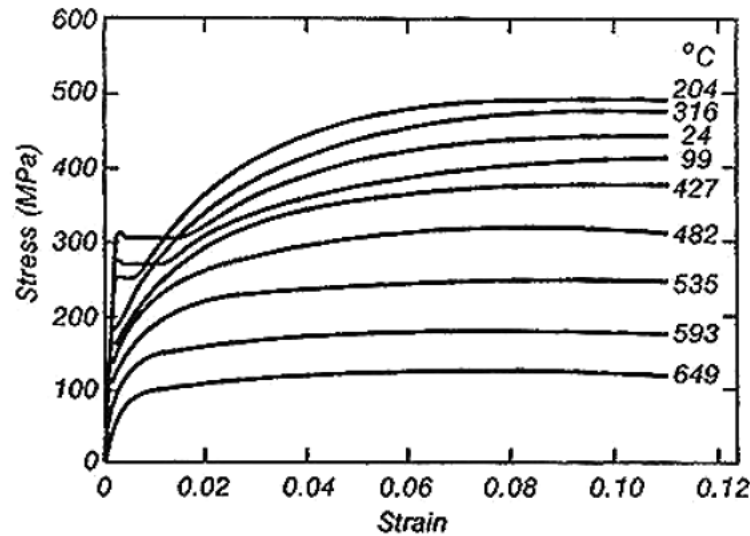


Figure 3.2.- Stress-strain curves of steel at elevated temperature (Buchanan, 2001).

It should be noted that in the stress-strain curves given in EN 1993-1-2 the creep strain is included in the curves.

### 3.3.2 Elastic modulus

The definition of the elastic modulus of an isotropic material is the tendency of an object to be deformed when a load is applied on it. It refers to the linear way of deformation, when an increase of stress generates an increase of deformation. It is hence, the slope of the elastic part of the materials stress-strain diagram. It is also related to the stiffness of the material, with higher elastic modulus, more difficult to deform the material. It is expressed as the stress per unit of strain and the normal units are  $\text{N/m}^2$  (Pa).

A bi-linear plastic material is used in this research with an elastic modulus of  $210 \times 10^9 \text{ N/m}^2$  and a shear modulus of  $79.3 \times 10^9 \text{ N/m}^2$ . Since in this research the steel is going to be evaluated with fire conditions, the Elastic modulus of steel will change according to Eurocodes and it is explained below.

### 3.3.3 Ultimate and yield strengths

In Eurocode (EN 1993-1-2, 2005) is shown the generalized relationship between the stress and the strain and it is shown in Figure 3.3. It is used to determine the resistances of steel such as tension, compression, bending moments and shear forces. In this research a bi-linear elastic-perfectly plastic material is used with the yield strength 355 MPa. Strain hardening and decay phase are not considered in the analysis method.

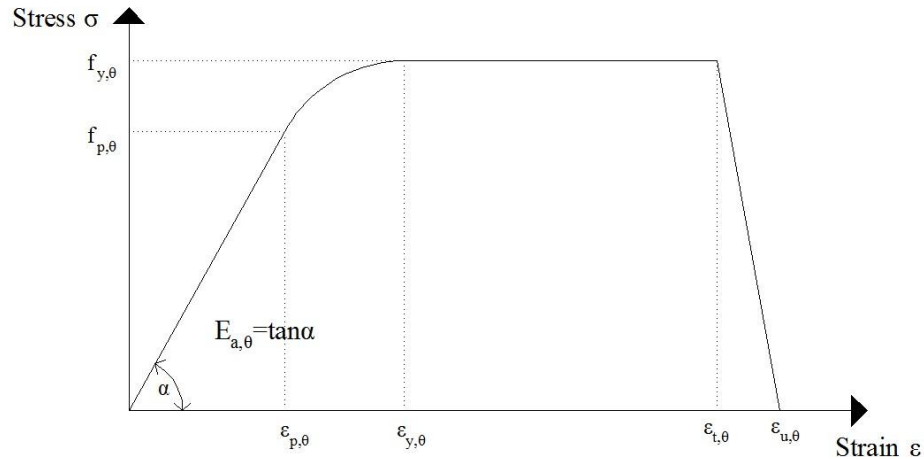


Figure 3.3.-Stress-strain diagram accepted in Eurocodes (EN 1993-1-2, 2005).

where

- $f_{y,θ}$  is effective yield strength;
- $f_{p,θ}$  is proportional limit;
- $E_{a,θ}$  is slope of the linear elastic range;
- $ε_{p,θ}$  is the strain at the proportional limit;
- $ε_{y,θ}$  is the yield strain;
- $ε_{t,θ}$  is the limiting strain for yield strength;
- $ε_{u,θ}$  is the ultimate strain.

In the Eurocode 3, is assumed that at the ambient temperature (20°C) the stress-strain relationship is bilinear and proportional limit  $f_p$  is equal to the yield strength  $f_y$ . It also assumes the absence of strain hardening.

In the Eurocode 3 are given the equations to calculate the steel behavior with temperature growing. Those equations are given in Table 3.1:

Table 3.1.-Stress-strain values proposed in Eurocodes (EN 1993-1-2, 2005).

Strain range	Stress $\sigma$	Tangent modulus
$\varepsilon \leq \varepsilon_{p,0}$	$\varepsilon E_{a,0}$	$E_{a,0}$
$\varepsilon_{p,0} < \varepsilon < \varepsilon_{y,0}$	$f_{p,0} - c + (b/a) [a^2 - (\varepsilon_{y,0} - \varepsilon)^2]^{0.5}$	$\frac{b(\varepsilon_{y,0} - \varepsilon)}{a [a^2 - (\varepsilon_{y,0} - \varepsilon)^2]^{0.5}}$
$\varepsilon_{y,0} \leq \varepsilon \leq \varepsilon_{u,0}$	$f_{y,0}$	0
$\varepsilon_{u,0} < \varepsilon < \varepsilon_{u,0}$	$f_{y,0} [1 - (\varepsilon - \varepsilon_{u,0}) / (\varepsilon_{u,0} - \varepsilon_{u,0})]$	-
$\varepsilon = \varepsilon_{u,0}$	0,00	-
Parameters	$\varepsilon_{p,0} = f_{p,0} / E_{a,0}$ $\varepsilon_{y,0} = 0,02$ $\varepsilon_{u,0} = 0,15$ $\varepsilon_{u,0} = 0,20$	
Functions	$a^2 = (\varepsilon_{y,0} - \varepsilon_{p,0})(\varepsilon_{y,0} - \varepsilon_{p,0} + c / E_{a,0})$ $b^2 = c (\varepsilon_{y,0} - \varepsilon_{p,0}) E_{a,0} + c^2$ $c = \frac{(f_{y,0} - f_{p,0})^2}{(\varepsilon_{y,0} - \varepsilon_{p,0}) E_{a,0} - 2(f_{y,0} - f_{p,0})}$	

As is mentioned before, steel mechanical properties vary with temperature. In order to check resistances in fire situations, there are some coefficients proposed to be used when considering the resistance of steel to each elevated temperature. Those reduction factors are shown in Table 3.2.

Table 3.2.- Reduction factors for the stees-strain of steel (EN 1993-1-2, 2005).

Steel temperature, $^{\circ}\text{C}$	$k_{y,\theta} = \frac{f_{y,\theta}}{f_y}$	$k_{E,\theta} = \frac{f_{a,\theta}}{E_a}$	$k_{p,\theta} = \frac{f_{p,\theta}}{f_y}$
20	1	1	1
100	1	1	1
200	1	0.9	0.807
300	1	0.8	0.613
400	1	0.7	0.42
500	0.78	0.6	0.36
600	0.47	0.31	0.18
700	0.29	0.13	0.075
800	0.11	0.09	0.05
900	0.06	0.0675	0.0375
1000	0.04	0.045	0.025
1100	0.02	0.0225	0.0125
1200	0	0	0

where

- $k_{y,\theta}$  is the reduction of yield strength of the steel;
- $k_{E,\theta}$  is the elastic limit reduction;
- $k_{p,\theta}$  is the proportional limit reduction.

The interpolation is accepted to calculate the intermediate values. In Figure 3.4 can be seen the graphical representation of the Table 3.2.

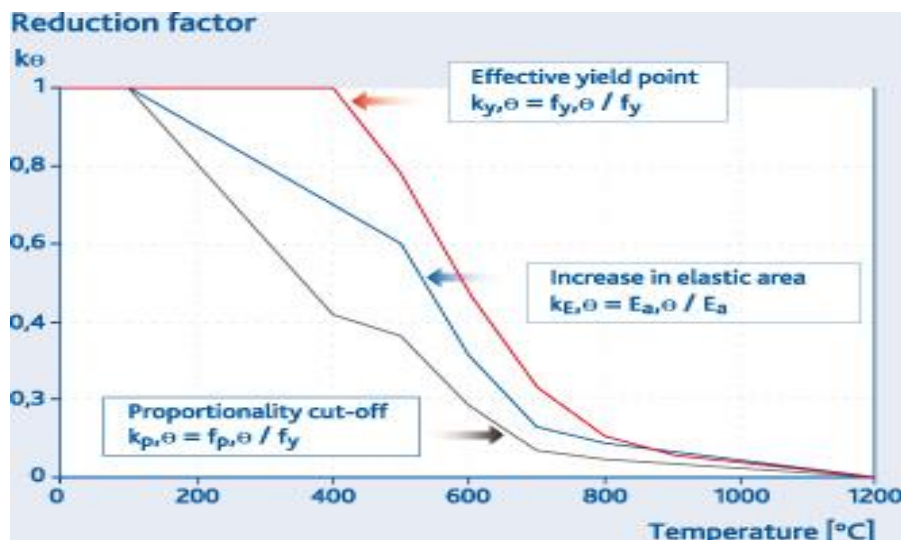


Figure 3.4.- Reduction factors for the stress-strain of steel (EN 1993-1-2, 2005).

In this figure it is easily shown how the yield limit is staying more constant with elevated temperatures and the proportional limit changes faster. In the middle of those is situated the Young's modulus.

Looking at it more closely, around 600 °C the yield strength of steel is about the half of the usual at 20°C. Then usually most of the deformation corresponds to the plastic region (proportionality constant is around the 80 % of ambient temperature). Therefore, the deformation will start to be highly significant, in some cases not acceptable.

## 3.4 Steel thermal properties

### 3.4.1 Density

In Eurocode 3 (EN 1993-1-2, 2005) the standard value for steel density is:

$$\rho = 7850 \text{ kg/ m}^3$$

For all the calculations it is going to be considered as independent of the temperature of the steel member but as it is commented before, the volume of steel pieces change with elevated temperatures.

### 3.4.2 Thermal conductivity

The usual value of steel thermal conductivity proposed in Eurocode 3 (EN 1993-1-2, 2005) is:

$$k = 54 \text{ W/mK}$$

However the thermal conductivity value varies with the temperature. In the Eurocode (EN 1993-1-2, 2005) is given:

- for  $20^\circ\text{C} \leq T_s \leq 800^\circ\text{C}$

$$k = 54 - 3.33 \times 10^{-2} T_s \quad (3.2a)$$

- for  $800^\circ\text{C} \leq T_s \leq 1200^\circ\text{C}$

$$k = 27.3 \quad (3.2b)$$

This variation of the thermal conductivity of steel is illustrated in the next Figure 3.5. For temperatures higher than  $1200^\circ\text{C}$  the value used for thermal conductivity is  $27.3 \text{ W/mK}$ .

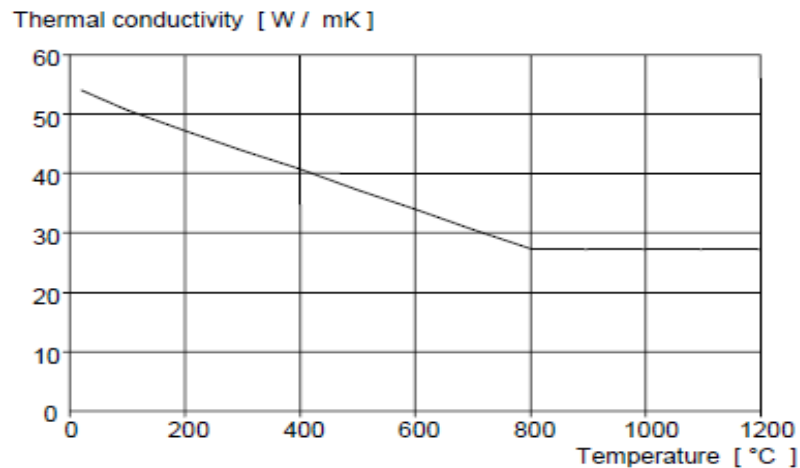


Figure 3.5.-Thermal conductivity as function of temperature (EN 1993-1-2, 2005).

### 3.4.3 Specific Heat

The specific heat of steel members is calculated using the following equations (EN 1993-1-2, 2005):

- for  $20^\circ\text{C} \leq \theta_a \leq 600^\circ\text{C}$

$$c_a = 425 + 7.73 \times 10^{-1} \theta_a - 1.69 \times 10^{-3} \theta_a^2 + 2.22 \times 10^{-6} \theta_a^3 \text{ J/kgK} \quad (3.3a)$$

- for  $600\text{ }^{\circ}\text{C} \leq \theta_a \leq 735\text{ }^{\circ}\text{C}$

$$c_a = 666 + \frac{13002}{738 - \theta_a} \text{ J/kgK} \quad (3.4b)$$

- for  $735\text{ }^{\circ}\text{C} \leq \theta_a \leq 900\text{ }^{\circ}\text{C}$

$$c_a = 545 + \frac{17820}{\theta_a - 731} \text{ J/kgK} \quad (3.5c)$$

- for  $900\text{ }^{\circ}\text{C} \leq \theta_a \leq 1200\text{ }^{\circ}\text{C}$

$$c_a = 650 \text{ J/kgK} \quad (3.6d)$$

where

- $\theta_a$  is the steel temperature.

In Figure 3.6 is shown the relationship between the specific heat and the temperature.

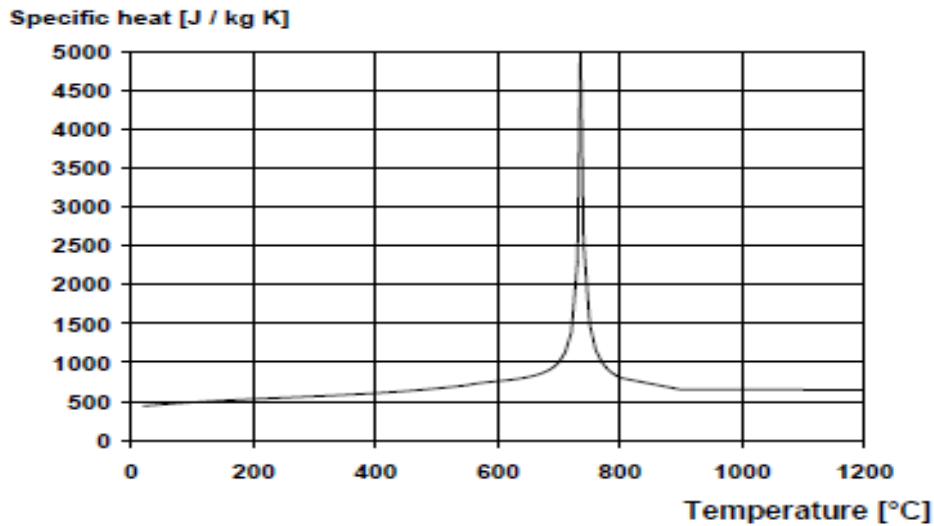


Figure 3.6.-Specific heat as function of temperature (EN 1993-1-2, 2005).

Also, the constant value for the specific heat can be used in Eurocodes (EN 1993-1-2, 2005) which is:

$$c_a = 600 \text{ J/kgK}$$

#### 3.4.4 Thermal expansion

As a first approximation, the change in length of members because of the thermal expansion capability of it happens due to the “linear expansion coefficient”. It is the fractional change in length by degree. With the consideration of an insignificant effect of the pressure, this coefficient is:

$$\alpha_L = \frac{1}{L} * \frac{dL}{dT} \quad (3.4)$$

where  $L$  is the length used for the measurement and  $dL/dT$  is the rate of change of the linear dimension by unit changed with the temperature.

As it is seen below (Figure 3.7) the expansion coefficient of steel vary with elevated temperatures.

Thermal elongation is temperature dependent and can be evaluated on the equations proposed in the Eurocode 3 (EN 1993-1-2, 2005).

- for  $20\text{ }^{\circ}\text{C} \leq \theta_a \leq 750\text{ }^{\circ}\text{C}$

$$\Delta L/L = 1,2 \times 10^{-5} \theta_a + 0,4 \times 10^{-8} \theta_a^2 - 2,416 \times 10^{-4} \quad (3.5a)$$

- for  $750\text{ }^{\circ}\text{C} \leq \theta_a \leq 860\text{ }^{\circ}\text{C}$

$$\Delta L/L = 1,1 \times 10^{-2} \quad (3.5b)$$

- for  $860\text{ }^{\circ}\text{C} \leq \theta_a \leq 1200\text{ }^{\circ}\text{C}$

$$\Delta L/L = 2 \times 10^{-5} \theta_a - 6,2 \times 10^{-3} \quad (3.5c)$$

where

- $L$  is the length at  $20\text{ }^{\circ}\text{C}$ ;
- $\Delta L$  is the thermal elongation;
- $\theta_a$  is the steel temperature.

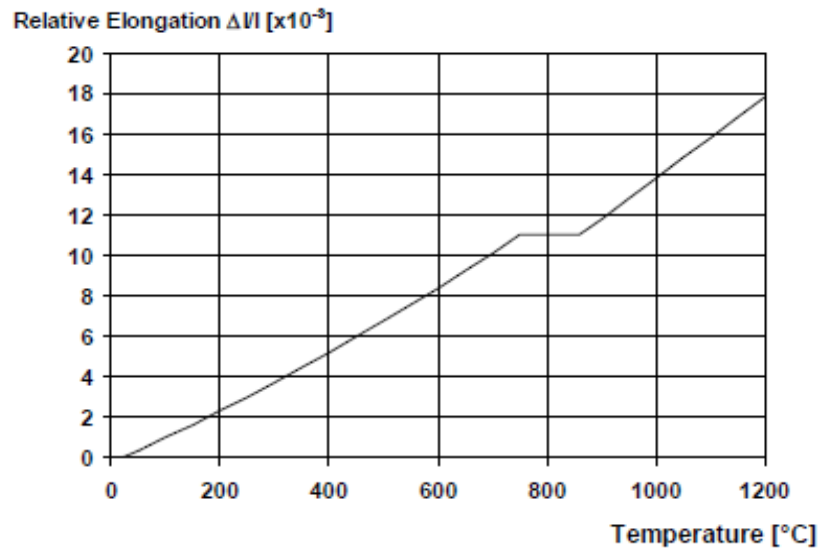


Figure 3.7.-Thermal elongation as function of temperature (EN 1993-1-2, 2005).

The properties proposed in EN 1993-1-2 are used in this study.

## 4. GEOMETRICAL AND ANALYSIS MODEL, BEAM MODEL

In this chapter the geometrical CAD model and the associated FEM for the truss used in this research is given. Also, the resistance check of each member is done in this chapter as well as the joint design and characteristics. The constraints are all requirements which are given in Eurocode 3 (EN 1993-1-1, 2005) (EN 1993-1-8, 2005).

### 4.1 Geometrical model

The starting point in this chapter is the geometry of the structure. The geometries of typical roofs are made with a Warren-typed truss with two non-parallel chords, sixteen braces and no verticals. The truss is a typical one span symmetric truss with an inclination of the upper chord (top chord) of 1:20 and a span of  $L=36$  m. (Heinisuo, Tiainen, & Jokinen, 2013), see Figure 4.1.

The truss connections are made with welded gap joints. There will be some eccentricities (eccentricity between the hinge of the braces and the chord) that are taken into account as studied later. Due to the symmetry of the structure, just one half of it is studied in this study. For braces, which are hinge ended members, and chords, which are continuous beams, Euler-Bernoulli beam elements are used (commented below). Cold-formed tubular members are used. Only cross-sections with a thickness bigger than 3 mm are considered due to the limitations in Eurocodes (EN 1993-1-8, 2005). In order to deal with the welded joints HEM1000 elements have been used to connect the chords with the braces. This rigid element will always be perpendicular to the direction of the chord avoiding inclined eccentricity (Heinisuo, Möttönen, Paloniemi, & Nevalainen, 1999). In order to study the structure with FEM in ABAQUS the mid line of each element will be displayed.

In Figure 4.1 the geometrical model used in the study is shown with the values of the lengths  $a_i$  explained in Figure 4.3.



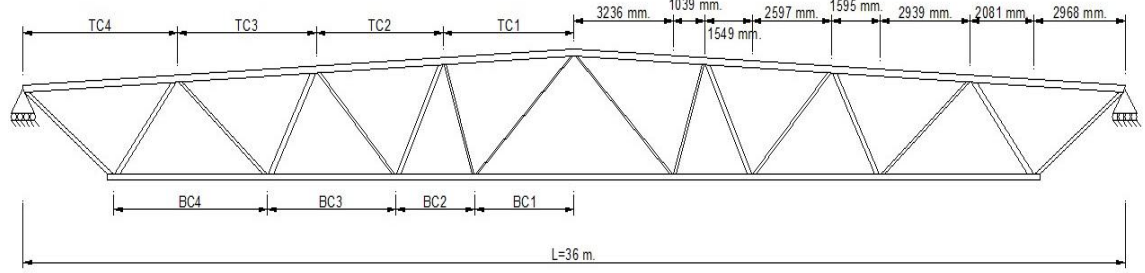


Figure 4.1.- Structure of the truss considered in the analysis.

The parameters needed to calculate the truss geometrical model (Figure 4.2) are given below.

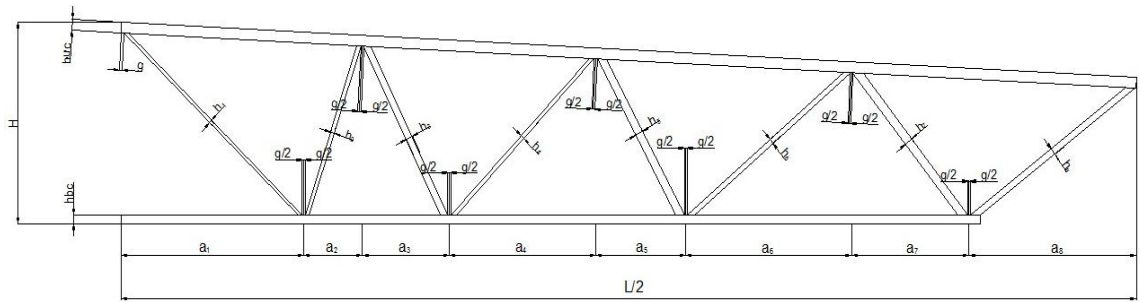


Figure 4.2.- Truss type considered, detailed.

The parameters needed are:

- The values of each  $a_i$  ( $i=1-8$ ) (Figure 4.2);
- The values of each  $h_i$  ( $i=1-8$ ) (Figure 4.3);
- Total length ( $L=36$  m);
- Total height ( $H=3269.79$  mm);
- Gap dimension ( $g=50$  mm);
- Inclination ( $\alpha=0.05$ );
- Chord profiles ( $h_{uc}=0.18$  m and  $h_{bc}=0.14$  m).

The values from the structure of our study are in Table 4.1:

Table 4.1.- Values of the joints location.

Braces	$a_i$ [mm]	$h_i$ [mm]
1	3236	50
2	1039	70
3	1540	120
4	2597	70
5	1595	140
6	2939	80
7	2081	140
8	2968	110

The geometrical entities (Heinisuo et al, 1999) that are needed to make the geometrical model are given in Figure 4.3.

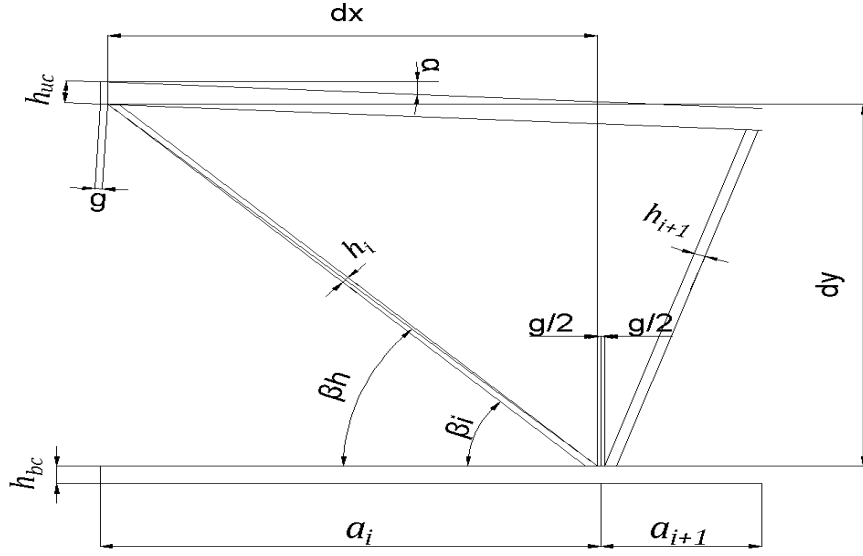


Figure 4.3.-Geometry of the truss.

The eccentricities of the joints have been calculated to avoid limitations of Eurocodes (EN 1993-1-8, 2005). At the support, the brace is connected directly to the chord without any eccentricity. This joint is studied normally case by case.

So, the values of each  $dx$ ,  $dy$ ,  $\beta_h$  and  $\beta_i$  (Figure 4.3) are calculated with the following equations (4.1)-(4.7) (Heinisuo, Tiainen, & Jokinen, 2013). With these values we are going to be able to get the position of the joint ( $e_x$  and  $e_y$ ) in the analysis model of study. Since the first joint is different, a distinction is going to be done between the first and the others not only in the geometry but also in the resistance check.

$$dx_1 = a_1 - g \times \cos \alpha - \frac{g}{2} \quad (4.1)$$

$$dx_i = a_i - \frac{g}{2} - g \times \frac{\cos \alpha}{2}, i=2-8. \quad (4.2)$$

$$dy_1 = H - h_{bc} - \frac{h_{uc}}{\cos \alpha} - g \times \sin \alpha \quad (4.3)$$

$$dy_i = H - h_{bc} - \frac{h_{uc}}{\cos \alpha} - \tan \alpha \times \left( \sum_1^i a_i - \frac{g}{2} \right), i=2, 4, 6, 8. \quad (4.4)$$

$$dy_i = H - h_{bc} - \frac{h_{uc}}{\cos \alpha} - \tan \alpha \times \left( \sum_1^{i-1} a_i - \frac{g}{2} \right), i=3, 5, 7. \quad (4.5)$$

The angle  $\beta_h$  can be calculated from distances  $dx$  and  $dy$ .

$$\beta_h = \arctg\left(\frac{dy}{dx}\right) \quad (4.6)$$

The  $\beta_i$  angle comes from a second order polynomial expression from which  $\beta_i > \beta_h$  is chosen.

$$\frac{hi^2 - dy^2}{\sin \beta_i^2} - 2 \times hi \times \cos \beta_h \times \frac{\sqrt{dx^2 + dy^2}}{\sin \beta_i^2} + dx^2 + dy^2 = 0 \quad (4.7)$$

At this moment, with all these values achieved, we can calculate the position of the joint with  $e_x$  (eccentricity perpendicular to the chord) and  $e_y$  (in the direction of the chord) (4.8-4.9). Here it is used the values of both braces that make the joint, and the value  $h_0$ , which is  $h_{uc}$  (upper chord) or  $h_{bc}$  (bottom chord). The angle  $\beta$  is  $\beta_i$

$$e_x = \left( \frac{h_1}{2 \times \sin \beta_1} + \frac{h_2}{2 \times \sin \beta_2} + g \right) \times \frac{\sin \beta_1 \times \sin \beta_2}{\sin(\beta_1 + \beta_2)} - \frac{h_0}{2} \quad (4.8)$$

$$e_y = \frac{g}{2} + \frac{h_2}{2 \times \sin(\beta_2 - \alpha)} - \frac{e_x + \frac{h_0}{2}}{tg(\beta_2 - \alpha)} \quad (4.9)$$

Applying all these expressions, and using our input data, we arrive to these values of eccentricities, given in Table 4.2 and in Figure 4.4 can be seen the truss studied in detail.

Table 4.2.- Geometrical features, truss studied.

Braces	$a_i$ [mm]	$h_i$ [mm]	$dx$ [mm]	$dy$ [mm]	$\beta_h$ [rad]	$\beta_i$ [rad]	$e_x$ [mm]	$e_y$ [mm]
1	3236	50	3161	2947	0.75	0.76	10	0
2	1039	70	0989	2736	1.22	1.24	19	32
3	1540	120	1490	2733	1.07	1.11	95	-10
4	2597	70	2547	2530	0.78	0.80	43	-35
5	1595	140	1545	2526	1.02	1.06	27	34
6	2939	80	2890	2303	0.67	0.69	39	-44
7	2081	140	2031	2299	0.84	0.89	10	28
8	2968	110	2943	2049	0.60	0.63	37	-28



where  $N_{Rd}$ ,  $M_{y,Rd}$  and  $M_{z,Rd}$  are the design values of the resistance depending on the cross section classification.

As a planar tubular truss axial forces, shear and bending moments are going to be present in the structure, so the interaction between those efforts is considered in this part. Also, as some beams are going to be in compression, the buckling is also studied.

As is mentioned before, no shear deformation is considered while using Euler-Bernoulli beam (Timoshenko & Goodier, 1951) elements, so besides the deflection, and in order to calculate the stresses, the forces and moments are used.

The steel grade used in the whole study is S355 in all the members. In this case, the resistances of the truss are going to be checked while loading on the upper chord with a load formed with a  $1\text{ kN/m}^2$  dead load and  $2\text{ kN/m}^2$  snow load in a roof width of  $5.4\text{ m}$ . With the ultimate state coefficients defined in the Finnish code, our load is:

$$(1.35 \times 1 + 1.5 \times 2) \times 5.4 = 23.5 \text{ kN/m} \quad (4.11)$$

In this checkout, we are considering  $\gamma_{M0}$  and  $\gamma_{M1}$  as 1.

For this part, the resistance check is done considering the support as a vertical restraint only. The horizontal displacement is allowed in the structure in both sides. The boundary condition at the middle of the truss (left side of our model) is a symmetric condition that allows the movement vertically. When the fire is considered different boundary conditions are studied for the supports. All of them are studied in chapter 6.

### 4.2.3 Geometrical features

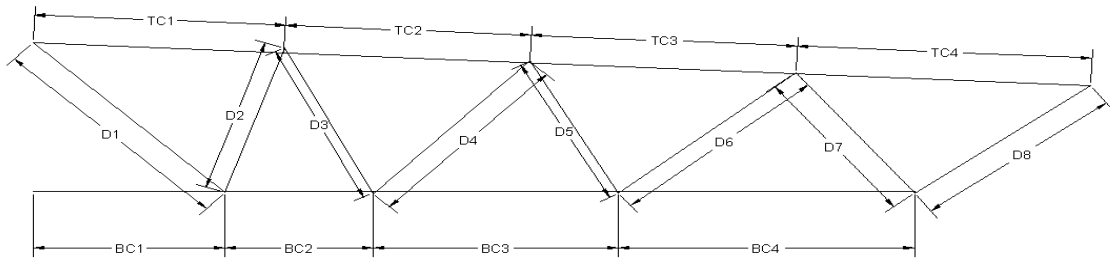


Figure 4.5.- Analysis model of the truss (joints eccentricities included).

For the members, cold-formed tubular sections are chosen. As the minimal thickness for the wall is  $3\text{ mm}$ , only thicker ones have been used. Using the denotation for each mem-

ber as in Figure 4.5, the profiles used for each member are given in Table 4.3 with the thickness ( $t$ ), height ( $h$ ) and width ( $b$ ):

Table 4.3.- Cross section features.

	$h$ [mm]	$b$ [mm]	$t$ [mm]
TC1	180	180	10
TC2	180	180	10
TC3	180	180	10
TC4	180	180	10
BC1	140	140	8
BC2	140	140	8
BC3	140	140	8
BC4	140	140	8
D1	50	50	3
D2	70	70	3
D3	120	120	4
D4	70	70	3
D5	140	140	5
D6	80	80	3
D7	140	140	5
D8	110	110	4

Geometrical characteristics of each profile are area, inertia and section modulus and they are calculated as given in Eurocode (EN 1993-1-1, 2005). All profiles must be class 1 or 2. Formulas needed to estimate those features are given as (4.12-4.14):

$$A = 2t(b + h - 2t) - (4 - \pi)(r_0^2 - r_i^2) \quad (4.12)$$

$$W_{pl,y} = \frac{bh^2}{4} - \frac{(b - 2t)(h - 2t)^2}{4} - 4(A_g h_g) + 4(A_\varepsilon h_\varepsilon) \quad (4.13)$$

$$I_y = \frac{bh^3}{12} - \frac{(b - 2t)(b - 2t)^3}{12} - 4(I_g + A_g h_g^2) + 4(I_\varepsilon + A_\varepsilon h_\varepsilon^2) \quad (4.14)$$

The corner radii used to calculate the geometrical features are calculated using the next instructions (EN 1993-1-1, 2005):

- If tube wall thickness  $t$  is smaller or equal to 6 mm then the outer radius of the corner  $r_0$  is 2 times the wall thickness;
- If the wall thickness is larger than 10 mm then the outer radius  $r_0$  is 3 times the wall thickness;
- In between it is 2.5 times the wall thickness.

The values for all these features are given in the next table (Table 4.4):

Table 4.4.-Member geometrical features.

	$A$ [mm <sup>2</sup> ]	$W_{pl,y}$ [mm <sup>2</sup> ]	$I_y$ [mm <sup>2</sup> ]	$r_o$ [mm]	$r_i$ [mm]
TC1	6456	403514	30167994	25	15
TC2	6456	403514	30167994	25	15
TC3	6456	403514	30167994	25	15
TC4	6456	403514	30167994	25	15
BC1	4004	194175	11267730	20	12
BC2	4004	194175	11267730	20	12
BC3	4004	194175	11267730	20	12
BC4	4004	194175	11267730	20	12
D1	540	9387	194671.4	6	3
D2	780	19415	575266.4	6	3
D3	1814	78326	4022759	8	4
D4	780	19415	575266.4	6	3
D5	2635	132303	7905593	10	5
D6	900	25779	878425.6	6	3
D7	2635	132303	7905593	10	5
D8	1654	65212	3059368	8	4

#### 4.2.4 Utilities of each member

Finally, for the members' resistance check, two formulas of utility (shear and combination of axial-bending moment) are going to be checked, and both must be under or equal to 1 (4.15)-(4.16). This check is done taking into account buckling resistance of members. The equations used are given in EN 1993-1-1 and commented in chapter 2.

$$\frac{Q_{ed}}{Q_{pl}} \leq 1 \quad (4.15)$$

$$\frac{N_{ed}}{\chi N_{pl}} + \frac{k_{yy} \times M_{y,ed}}{M_{pl}} \leq 1 \quad (4.16)$$

In the next table (Table 4.5) it is shown the values of the utilities calculated for each member and all of them are under or equal to 1, so the structure is feasible in this case. The forces and the moments of the members are calculated using the linear elastic analysis theory using the analysis model of Figure 4.5 with the joint eccentricities of Table 4.2.

Table 4.5.-Members utilities in linear situation.

Member	$Q$	$N+M$
TC1	0.08	0.98
TC2	0.08	0.96
TC3	0.09	0.87
TC4	0.11	0.71
BC1	0.00	0.89
BC2	0.00	0.96
BC3	0.00	0.83
BC4	0.02	0.95
D1	-	0.07
D2	-	0.09
D3	-	0.25
D4	-	0.46
D5	-	0.33
D6	-	1.00
D7	-	0.68
D8	-	1.00

### 4.3 Joint resistance check

Some different parameters of the joints have a very important effect on its resistance. This resistance is especially dependent on the joint type of the structure and the type of the force that acts on the joint (tension, compression or moment.) In this paper, welded joints are studied using the rules of Eurocode 3 (EN 1993-1-8, 2005).



The most usual failure modes of a joint depend on the geometric parameter of the joint, as well. Those failure modes are:

- Chord face failure;
- Chord side wall failure;
- Chord shear failure;
- Chord punching shear;
- Bracing effective width;
- Chord or bracing local buckling;
- Shear of overlapping bracing.

In this case only some of them are studied (validity of the welded joints studied below). The checkout of the utilities of each K-joint is divided in 5 different checks (4 checks of the braces and 1 check for the chord) and the gap conditions. The data needed for each joint is:

- $b_0$  and  $t_0$  of the chord;
- $b_1, t_1, \theta_1, b_2, t_2$  and  $\theta_2$  of both braces;
- $g$ .

In Figure 4.6 are shown the joints that are studied in this part of the report. First and last joint are not consider in this analysis because they have different behavior than the other 7. In this part is only explained the way to check the joints from 1 to 7.

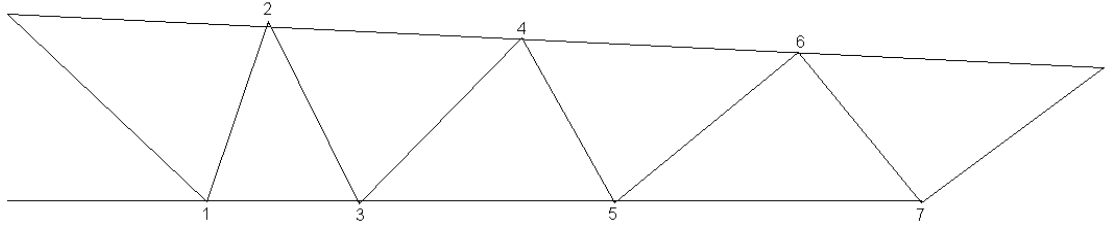


Figure 4.6.-Joints whose failure mode is checked.

The resistances checked at joints (Figure 4.6) in this study are calculated with the equations that are given below ((4.17)-(4.29):

- Chord face failure:

$$N_{Rd,i} = \frac{(8.9 \times k_n \times f_{y0} \times t_0^2 \times \sqrt{\gamma})}{\sin \theta_1} \times \frac{(b_1 + b_2)}{2b_0}, i=1, 2; \quad (4.17)$$

- Chord shear:

$$N_{Rd,i} = \frac{f_y \times A_v}{\sqrt{3} \sin \theta_i}, i=1, 2; \quad (4.18)$$

- Chord face punching shear:

$$N_{Rd,i} = \frac{f_y \times t_o}{\sqrt{3} \sin \theta_i} \times \left( \frac{2b_i}{\sin \theta_i} + b_i + b_{ep,i} \right), i=1, 2; \quad (4.19)$$

- Brace failure:

$$N_{Rd,i} = f_y \times t_i \times (2b_i - 4t_i + b_i + b_{eff,i}), i=1, 2; \quad (4.20)$$

- Chords shear:

$$N_{Rd,g,o} = (A_o - A_{v,o})f_y + A_{v,o} \times f_y \sqrt{1 - \left( \frac{V_{ed}}{V_{pl}} \right)^2}, i=1, 2. \quad (4.21)$$

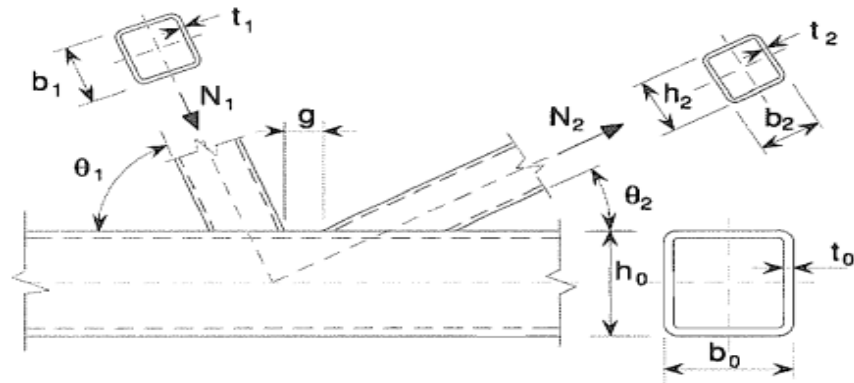


Figure 4.7.- K and N gap joint (EN 1993-1-8, 2005).

It is also needed for braces and shear calculation:

$$b_{ep,i} = \min \left\{ \frac{10b_i}{b_o/t_o}; b_i \right\} \quad (4.22)$$

$$b_{eff,i} = \min \left\{ \frac{10f_y \times t_o \times b_i}{\left( \frac{b_o}{t_o} \right) \times t_i \times f_y}, b_i \right\} \quad (4.23)$$

$$A_v = (2b_o + \alpha b_o)t_o \quad (4.24)$$

$$V_{pl} = A_v \times \frac{f_y}{\sqrt{3}} \quad (4.25)$$

where

$$\alpha = \sqrt{\frac{1}{1 + \frac{4g^2}{3t_o^2}}} \quad (4.26)$$

For compression:

$$k_n = 1.3 - 0.4 \frac{\frac{N_{ed}}{a_o \times f_y} + \frac{M_{ed}}{W_{pl} \times f_y}}{\beta} \quad (4.27)$$

Consequently the utilities of K-joints are calculated as:

- Utility of the brace:

$$Max \frac{N_{i,Ed}}{N_{i,Rd}} \leq 1, i=1, 2; \quad (4.28)$$

- Utility of the chord:

$$\frac{N_{0,Ed}}{N_{0,gap,Rd}} \leq 1, i=1, 2. \quad (4.29)$$

After those calculations we must check if every of the resistances ( $N_{Rd}$ ) are equal or higher to the maximum axial force in each element ( $N_{Ed}$ ) and the moments ( $M_{Ed}$ ) taken at both sides of the joint. And these are the values achieved, see Table 4.6. The forces and the moments are calculated as for the member checks before.

Table 4.6. - Joints studied, utilities calculated [kN].

	Joint 1	Joint 2	Joint 3	Joint 4	Joint 5	Joint 6	Joint 7
Chord failure <sub>1</sub>	270.30	406.92	565.01	515.58	734.28	579.58	890.01
Chord Shear <sub>1</sub>	517.34	917.88	682.96	940.24	766.54	1072.38	817.61
Chord face punching shear <sub>1</sub>	445.43	1084.74	695.91	1315.18	962.48	1626.82	1474.17
Brace failure <sub>1</sub>	285.42	658.88	285.42	958.5	328.02	958.5	602.08
Force (N <sub>ed</sub> )	9.13	103.86	128.35	225.3	319.69	462.6	587.51
<b>Utility</b>	<b>0.03</b>	<b>0.25</b>	<b>0.45</b>	<b>0.43</b>	<b>0.97</b>	<b>0.79</b>	<b>0.97</b>
Chord failure <sub>2</sub>	371.39	368.66	453.10	583.78	535.99	638.73	685.52
Chord Shear <sub>2</sub>	710.80	831.58	547.69	1064.62	559.54	1181.82	629.76
Chord face punching shear <sub>2</sub>	530.82	541.16	835.59	803.83	1008.57	1090.63	1219.50
Brace failure <sub>2</sub>	200.22	285.42	658.88	285.42	937.2	328.02	937.2
Force(N <sub>ed</sub> )	13.8	9.13	103.86	128.35	225.3	319.69	462.6
<b>Utility</b>	<b>0.06</b>	<b>0.03</b>	<b>0.23</b>	<b>0.45</b>	<b>0.42</b>	<b>0.97</b>	<b>0.73</b>
N <sub>Rd,g,0</sub>	1421.50	2289.20	1421.50	2289.27	1421.50	2288.42	1421.39
<b>Utility</b>	<b>0.86</b>	<b>0.54</b>	<b>0.88</b>	<b>0.53</b>	<b>0.78</b>	<b>0.44</b>	<b>0.53</b>

It has been only done the resistance check for some failure modes because the geometry of the joints is within the range of validity for welded joints given in Eurocode (EN 1993-1-8, 2005). It will be necessary at that point to check the constraints dealing with gap and ratios of dimension of the chords and the braces. We use those formulas ((4.30)-(4.34) in order to check this:

$$\beta \leq 1 \quad (4.30)$$

$$\frac{b_i}{b_o} \leq 1 \quad (4.31)$$

$$g \geq t_1 + t_2 \quad (4.32)$$

$$\frac{g}{b_o} \geq 0.5(1 - \beta) \quad (4.33)$$

$$\frac{g}{b_o} \leq 1.5(1 - \beta) \quad (4.34)$$

If  $g/b_o \geq 0.5(1 - \beta)$  and  $g \geq t_1 + t_2$  then the K-joint is treated as two separate T-joints.

The real amount of constrains for the truss are (Mela, Heinisuo, & Tiainen, 2012):

- Utilities for interaction of axial force bending moment, 4 members at both chord, 8 members at braces, 16 constraints;
- 16 constraints for shear resistances of the members;
- Utilities of K-joints, 7 joints,  $7 \times 4 + 7 \times 1 = 35$ ;
- Angles 16 constraints, checked also support and top joints;
- Cross-section classes  $3 \times 7 = 21$ ;
- Geometrical constraints  $2 \times 7 = 14$ ;
- Gaps and dimensions  $6 \times 7 = 42$ ;
- Total amount 160 constraints.

#### 4.4 Other constraints

As is mentioned before, there are some constraints dealing with the angles of the braces that are slightly different in the analytical model compared with the geometrical model. The models have to fulfill the angles limitations, geometrical constraints and cross-section class required. Both sides of the joint will be studied in this part separately.

In some check, first and last joint will not be studied because they are studied case by case, and in this paper they are not taken into consideration.

- Angles between braces

In this part angles achieved with Matlab/Excel (analysis model) should be compared with the AutoCAD values (geometrical model) and take the most unfavorable and check with the following equation:

$$\frac{30}{\phi} \leq 1 \quad (4.35)$$

- Geometrical constrains

In this case, first and last joint are not considered, results are given in Table 4.7. The limitations given in Eurocode 3 (EN 1993-1-8, 2005) is given below:

$$\max \left\{ \frac{0.35}{\frac{b_i}{b_o}}; \frac{0.1 + \frac{0.01b_o}{t_o}}{\frac{b_i}{b_o}} \right\} \leq 1 \quad (4.36)$$

Table 4.7.- Geometrical constraints of joints check (two per joint 1-7).

GEOMETRICAL CONSTRAINT	$b_0$ [mm]	$t_0$ [mm]	$b_1$ [mm]	$t$ [mm]	$r$ [mm]	Constraint
1	140	8	50	3	6	0.98
2	140	8	70	3	6	0.7
3	180	10	70	3	6	0.9
4	180	10	120	4	8	0.52
5	140	8	120	4	8	0.41
6	140	8	70	3	6	0.7
7	180	10	70	3	6	0.9
8	180	10	140	5	10	0.45
9	140	8	140	5	10	0.35
10	140	8	80	3	6	0.61
11	180	10	80	3	6	0.78
12	180	10	140	5	10	0.45
13	140	8	140	5	10	0.35
14	140	8	110	4	8	0.44

- Cross-section class

This check is oriented to both chords and compressed braces. The results are shown in

Table 4.8:

$$\frac{b - 2r_o}{38t \sqrt{\frac{235}{f_y}}} \leq 1 \quad (4.37)$$

Table 4.8.- Cross-section class of members check.

	$b_1$	$b_2$	$b_3$	$b_4$	$b_5$	$b_6$	$b_7$	$b_8$
Cross-section class Diagonal	0.41	0.6	0.84	0.6	0.77	0.73	0.77	0.8
	upper lower							
Cross-section class Chord	0.42	0.4						

It can be concluded that the truss can resist the uniform load of 23.5 kN/m in the ambient conditions with respect to all the requirements of the Eurocodes (EN 1993-1-1, 2005) (EN 1993-1-8, 2005) dealing with the members and the joints.

## 5. ABAQUS DESIGN AND HEAT TRANSFER ANALYSIS

### 5.1 Abaqus design

#### 5.1.1 General

There are several computer programs that make possible the structural analysis under fire consideration. In this sub-chapter, a brief overall description of ABAQUS FEA is done. In this research ABAQUS/Standard has been used. It is a general-purpose Finite-Element analyzer that employs a traditional integration system. All of this information is extracted from ABAQUS analysis user's manual (Simulia, 2011). Many options are available in this software that makes possible the analysis of complex structural situations.

Thermal analysis is performed before ABAQUS analysis and the obtained uniform temperatures are used in the structural analysis as an input to ABAQUS, using the ODB file of the first model (heat transfer) (Cedeno, Varma, & Gore, 2006).

#### 5.1.2 Parts

The ABAQUS model is divided into nine different parts with different features considered. The chords and the rigid elements used to describe the welded connections are considered as one part. Each brace of the structure is another part in the ABAQUS model. This detachment has been done in order to define different elements between them and avoid creating point that not exists in the real structure. Also, the consideration of a continuous beam element in the chord is taken into account. The material properties of each member have already been defined in the chapter 3.

The merge of all the parts have been made using MPC (multi-point constraints) pinned in the joints in order to model the hinges. In Figure 5.1 can be seen what the joint model looks. In this study the rotational stiffness in Figure 5.1 is supposed to be zero, meaning that the braces are hinge ended members. The small eccentricity element, which is perpendicular to the chord, is modeled as a rigid link. The chord is modeled as continuous over the joint.



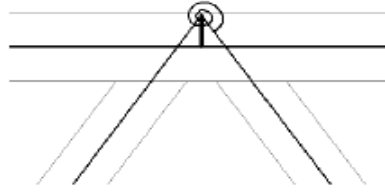


Figure 5.1.- Truss connections model (Boel, 2010).

### 5.1.3 Type of elements

Beam elements are the selected options to model the structure. Since two different models are studied in this research (heat transfer analysis and a static analysis with the cross-section temperatures previously calculated) different elements are used. In the heat analysis DC1D2 elements have been selected for the braces that model the elements (hinged at both ends). In the static analysis both braces and chords are modeled as beam elements (B33) taking into account Euler-Bernoulli theory without shear deformation of members. Each members is divided into 8/10 elements (minimum element length 0.5 m) and the element used to model the eccentricities of the welded joints has the properties of HEM1000. No temperature input is considered in these small members.

### 5.1.4 Type of section

In order to get a closer behavior of the structure arbitrary profiles are calculated in ABAQUS. With this option accuracy enough will be achieved and the cross section properties will be very similar to the real. With this option plasticity consideration and the fire behavior will be also more genuine.

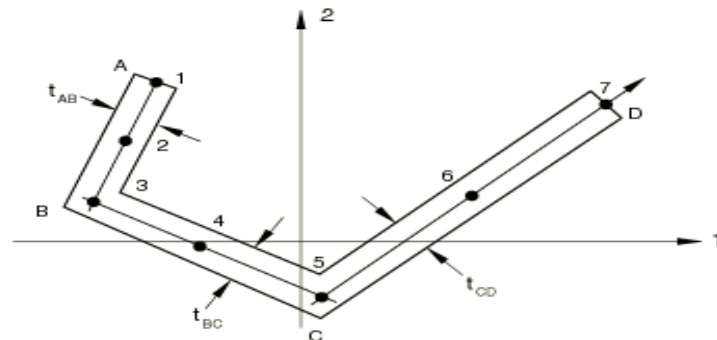


Figure 5.2.-Example of an arbitrary section, ABAQUS (Simulia, 2011).

The section is defined by specifying points in the thin-walled cross-section of the members; these points are then linked by straight line segments, each of which is integrated numerically along the axis of the section so that the section can be used together with non-linear material behavior. An independent thickness is associated with each of the segments making up the arbitrary section. Warping effects are included when an arbitrary section is used with open-section beam elements.

In order to achieve this section approximation (Figure 5.3) in this study has been used the method studied by J. Kukkonen and M. Heinisuo (Kukkonen, Heinisuo, & Toumala, 2005), where they studied the similarities and relations in area and inertia between the real profile and some approximations, so we will be able to model the real section with enough accuracy and giving a reliable area and moment of inertia of the whole section.

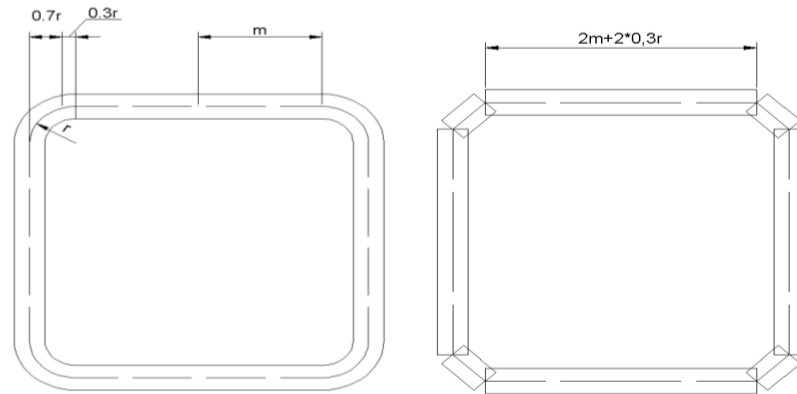


Figure 5.3.-Real cross-section and section studied, ABAQUS.

On the other hand, as we want to study the structure with fire as well, we need to know how this type of profile will work in fire. With beam sections integrated during the analysis with temperature and predefined fields at specific points, the temperatures should be given at each of the points shown in Figure 5.4:

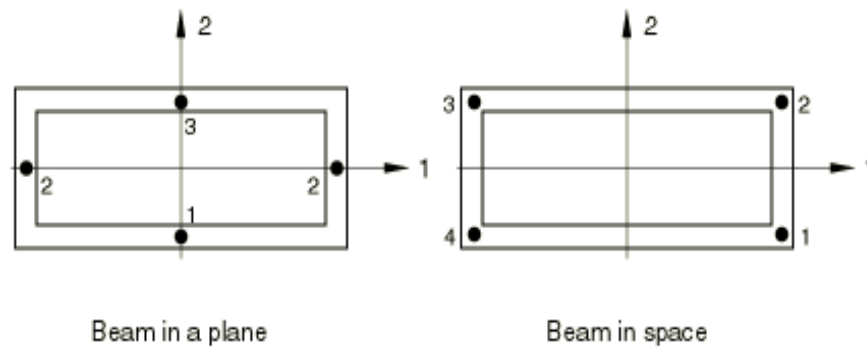


Figure 5.4.-Points where temperature is given in ABAQUS (Simulia, 2011).

### 5.1.5 Boundary conditions.

Boundary conditions implemented in the model are trying to simulate the support conditions on the one-span symmetric tubular truss. Due to this symmetry, only half of the truss is modeled in the program ABAQUS with a symmetry boundary condition in the middle of the truss (left side of the truss modeled). With this boundary condition,  $X$  displacement and  $\theta_y$  and  $\theta_z$  rotations are restrained. The coordinate system is shown in Figure 5.5.

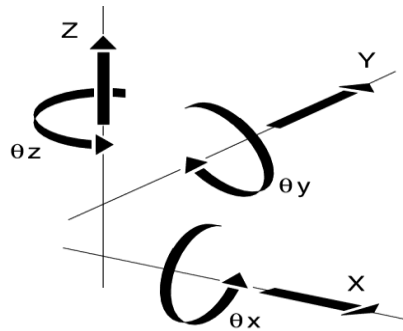


Figure 5.5.-Coordinate system used in ABAQUS (Simulia, 2011).

The external condition varies depending on the part of the research. Three different types of support conditions at the end support of the truss will be evaluated in the thesis and compared in order to know the differences on the behavior of a same truss in different conditions (Seputro, 2006). One statically determinate and two non-statically determined trusses will be studied.

#### 5.1.5.1 Roller condition (model 1):

In this model, the structure is simply supported and member's forces can be solved just with equilibrium if the truss is "internally" statically determinate. At both sides, there is not going to be any bending moment and the joint is modeled with a hinge and the horizontal movement is not going to be constrained. However, the ideal roller condition does not really exist. It is known that the real behavior of this truss is going to be between model 1 and model 2.

#### 5.1.5.2 Pin condition (model 2):

With pin at the ends of the truss the horizontal displacement is fully restrained. This support is also free to rotate, but the truss is not statically determined. This type is more common in real structures than model 1.

#### 5.1.5.3 Fix condition (model 3):

In this last case, the structure is going to be fully restrained; it will behave as an embedded model. In this case both rotation and horizontal displacement at the ends of the truss are restrained, so it is the most restricted model studied. In this case the bottom chord is extended to the support.

In all cases the truss is as it was in the case considered above. The members and the joints can resist the loads in model 1.

### 5.1.6 Spring consideration

The idea of a pin or a roller condition in a structure is very ideal in real structures. Even in simply supported conditions (model 1) of a structure, there is usually a little stiffness in the structure that restrains the horizontal movement. A spring should be considered in this case, in order to model the stiffness in roller conditions. If a spring is used in the design of the support conditions, an intermediate case between the roller case and the pin case is studied. It would be the behavior of a structure with a relative spring stiffness in this support of 20% or less, somewhere into the roller and the pin truss boundary conditions.

The value for this axial stiffness should be calculated with the following equation (5.1), considering the effect of the column supporting the truss:

$$K = AE/L \quad (5.1)$$

where  $K$  is the stiffness of the spring,  $E$  is the Young modulus,  $A$  is the area and  $L$  is the length of the spring.

This situation is only valid in the case of an isolated structure. If there is any kind of contact with other adjacent structure this hypothesis should be changed. In this research, in order to facilitate the calculations, we will take on one hand the hypothesis of only block the displacement  $Z$  (model 1) and axially blocked on the other hand (model 2).

### 5.1.7 Analysis procedure

There are two major models in analysis of structures at elevated temperatures using ABAQUS, the heat transfer model done before, and the static and mechanical analysis where the temperatures calculated in the first model are introduced as an input in the second model. The input files of ABAQUS consist of .ODB files with the values of the growing temperatures. This heat transfer analysis is explained in the next sub-chapter.

## 5.2 Heat transfer analysis

### 5.2.1 Fire resistance

After the resistance check done in chapter 4, next step is to introduce fire conditions in the analysis. The resistance of a structure to the fire is the ability of a building to resist the fire and the growing of temperature. This fire resistance is in this study calculated using the standard ISO fire where temperature grows with time in a certain speed that is discussed in this section.

The steel subjected to elevated temperatures suffers very important variations at all of its properties which are seen in Figure 5.6 and widely commented in Chapter 3.

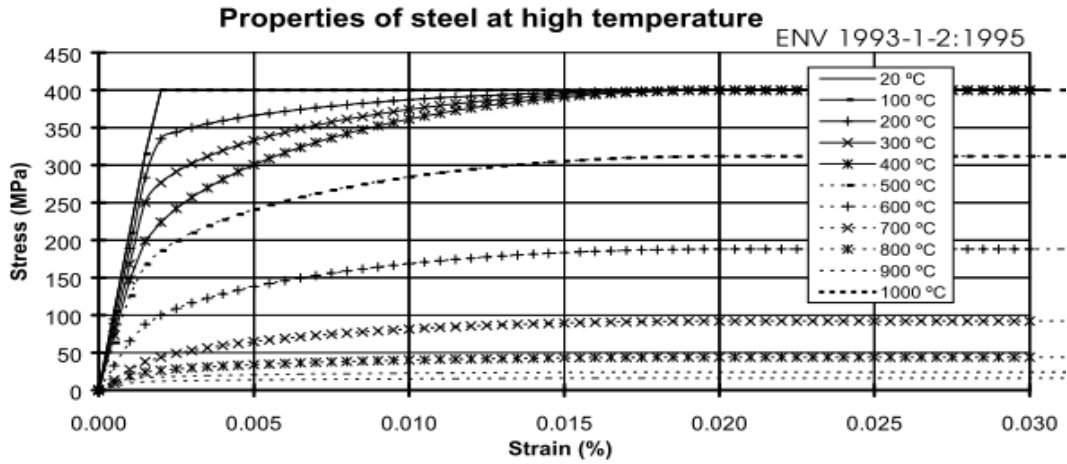


Figure 5.6.- Steel properties variation with temperature growing (EN 1993-1-2, 2005).

The normal calculation temperature for steel is 20 °C, so all the properties will be expressed over the value of it.

The fire resistance of any structure is also pretty related to the grade of hyperstaticity of the structure and if it is statically determined or non-statically determined since the thermal expansion and the heat could create different axial forces or help to create plastic hinges and redistribute the resistances of the structure.

### 5.2.2 ISO 834 fire

The steel temperatures  $T_s$  can be calculated by using the Equation 5.2 (EN 1993-1-2, 2005):

$$\Delta T_s = \frac{A_m/V}{\rho \cdot c} \times \left( \varepsilon \cdot \sigma \cdot (T_g^4 - T_s^4) + h \cdot (T_g - T_s) \right) \Delta t \quad (5.2)$$

where

- $V$  is the volume of the member per unit length [ $\text{m}^3$ ];
- $\rho$  is the density of the material [ $\text{kg}/\text{m}^3$ ];
- $c$  is the specific heat of the member [ $\text{Ws}/\text{kgK}$ ];
- $\Delta T_s (K)$  is the temperature change of the member during the time step (5sec);
- $\varepsilon$  is the emissivity of the surface of the member (0.7);
- $\sigma$  is the Stefan-Boltzmann constant  $5.67 \times 10^{-8} \text{W}/\text{m}^2 \text{K}^4$ ;
- $h$  is the convective heat transfer  $25 \text{ W}/\text{mK}$ ;
- $A_m$  is surface area per unit length of the cross-section [ $\text{m}^2$ ];
- $A_m/V$  in this case  $1/t(1/m)$ , where  $t$  is the wall thickness [ $\text{m}$ ] of the tube.

The temperature of the gas is calculated using ISO 834 standard fire, and with the next equation:

$$T_g = 20 + 345 \log_{10}(8t + 1) \quad (5.3)$$

where  $T_g$  the temperature of the gas and  $t$  is the time in minutes. The time-temperature relationship is shown in Figure 5.7.

The idea in this case, is to put temperatures growing in each member during a certain time and to see the evolution of the features of our truss. In Figure 5.7 is also shown the temperatures for all the different members that are used in the studied truss. The temperatures of the members are calculated using Equations 5.2 and 5.3.

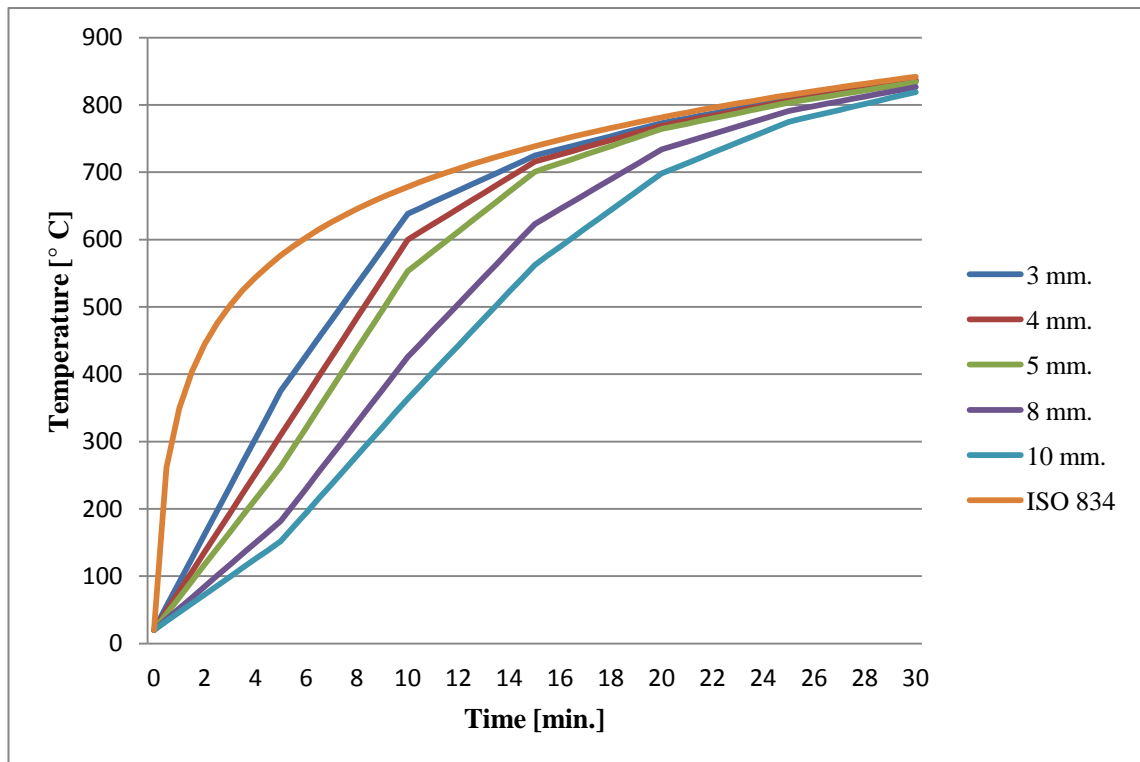


Figure 5.7.-Temperature of gas and members in ISO fire.

The standard temperature-time curve does not take into account factors such as different fuels, ventilation openings, compartment differences, differences in thermal properties of the boundary and the fact that the fire at some stage will decline.

In this analysis has been performed a linear and non-linear analysis with a constant load over time, but with the variation of temperatures. These variations go from 20 to 1200 °C, but growing with different speed depending on the shape of the member. The changes which we will study are the difference between ambient temperature, temperature after half an hour (R30) and after an hour (R60) in the whole truss.

However, since the truss structure is not going to stand more than 30 minutes in any case, the temperature is only shown until 30 min.

### 5.2.3 Members analysis

Since we are work with standard fire consideration, the verification method is going to be done using EN 1993-1-2. The effect of the actions will be determined for time=0 using combination factors  $\psi_{1,1}$  and  $\psi_{2,1}$  (EN 1993, Käsikirja, 2012) and shown in Figure 5.8

$$E_{d,fi} = \eta_{fi} E_d \quad (5.4)$$

where  $E_d$  is the design value for the corresponding force or moment for normal temperature design and  $\eta_{fi}$  is the reduction factor in fire situation.

The reduction factor  $\eta_{fi}$  (Ruukki, 2012) is calculated as below:

$$\eta_{fi} = \frac{G_k + \psi_{fi} Q_{k,1}}{\gamma_G G_k + \gamma_{Q,1} Q_{k,1}} \quad (5.5)$$

where

- $Q_{k,1}$  is characteristic value of the leading variable action;
- $G_k$  is characteristic value of a permanent action;
- $\gamma_G$  is the partial factor for permanent actions;
- $\gamma_{Q,1}$  is the partial factor for variable action 1.

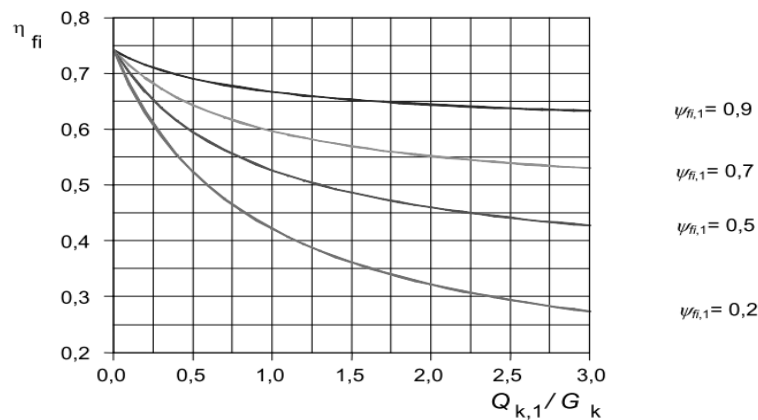


Figure 5.8.- Reduction factor  $\eta_{fi}$  variation with the load ratio  $Q_{k,1}/G_k$  (Ruukki, 2012)

Since it is an analysis in elevated temperatures with standard fire, the final analysis has been finished with approximately a 35 % of the real load, so it has been completed with an uniform vertical load at the top chord of 8.225 kN/m which is the design load using the recommended load factors in fire (Ruukki, 2012) (EN 1993-1-2, 2005) for the dead load 1.0 and for the snow load 0.3.

## 6. ANALYSES FOR DIFFERENT SUPPORT CONDITIONS

### 6.1 Introduction

In this chapter we are going to show the variations on the forces, moments, deflection and resistances of a heated structure under three different boundary conditions at the end support and the failure mechanisms as well. Both linear and non-linear theory is considered at all the models and their behavior is compared between them. The first subchapter is describing the stress resultant (forces and moments) and reactions generated in the structure (different models) without fire and in linear static case. In the next subchapters the study of each model is shown separately and a description of their behavior is also given. For structures with a rotation constraint in the support, bending moments and axial forces will be shown, while the temperature is growing.

It is known that the real structure would be supported by a column, so neither of the models is going to be totally correct. As it is discussed in chapter 5, the more approximate model would be the first case, a simply supported structure with a roller in the support (free to move laterally) but constrained with a spring with certain stiffness. The result will be extrapolated from the results and discussion of the three models will be done.

#### 6.1.1 Sign convention

When a negative deflection is given, the displacements of the truss are downwards. For axial forces, the convention used is negative for beams in tension and positive for compressive members. For bending moments the convention used is negative when the tension is below the center of the member and positive when it is above. This convention will be used in all analyses.

### 6.2 Stress resultants without fire

In this subchapter stress resultants of all models are given, discussed and compared using only the linear theory and without fire. Then it is possible to compare the models in ambient temperature, and also compare the evolution of each model when the temperature is acting. In this case, the reduced load of 8.225 kN/m is considered since the resistance check of all the members have already been done in chapter 4 with the real load of 23.5 kN/m. Model 1 and model 2 (both simply supported structures) are going to be



compared more between them and model 3 (fully restrained) will be studied apart from the other two. The stress resultants are given in Table 6.1 and Table 6.2.

As the table shows, both shear and bending moments are very similar in both cases since the whole structure behaves as a simply supported beam. However, with the axial forces there are quite different results, especially in the compression of the top chord, where the axial force in last part of it is near to zero in the axial restrained model (model 2). The explanation of it comes from the direction of the reaction force generated in the support, which should be almost parallel to the direction of the last brace (D8). So, it can be seen the higher values of axial on the braces of the second model due to the redistribution of the load between them.

It is also interesting to mention that the axial forces in the first two braces, brace 1 and 2 (D1 and D2 in Table 6.1), have different signs due to the value of the ‘reaction’ in the left part of the half truss (mid span) Anyway, those reactions are not going to affect to the truss behavior too much.

If the model 3 is compared with the simply supported structures (model 1 and 2) some bigger differences can be appreciate. Shear forces and moments continue being about the same, but there is even more changes in axial forces. The resultant in the top chord changes from a smaller compressive force in the beginning of it (TC1) and continues changing into a tensile effort in the last part (TC4). The bottom chord is also behaving quite different; it is also in tension in the first part (BC1, BC2 and BC3) but change into compression in the last part (BC 4 and BC5). It is due to the different way of distribute the load since it is a structure with the rotation restrained in both supports. It is almost totally fix structure, so the moment diagram of the whole structure is different from the simply supported models (model 1 and 2). Those resultants are given in Table 6.2.

Table 6.1.- Stress resultants without fire using linear theory, model 1 and 2.

	MODEL 1				MODEL 2			
	$N$ [kN]	$Q$ [kN]	$M_{max}$ [kNm]	$M_{min}$ [kNm]	$N$ [kN]	$Q$ [kN]	$M_{max}$ [kNm]	$M_{min}$ [kNm]
<b>TC1</b>	437.59	16.26	7.77	-8.28	306.75	15.87	6.77	-8.55
<b>TC2</b>	422.39	15.73	6.37	-8.55	285.82	16.04	7.39	-8.19
<b>TC3</b>	353.32	18.33	13.16	-7.09	205.69	18.36	13.43	-6.89
<b>TC4</b>	165.40	22.59	14.50	-16.38	1.83	22.68	14.95	-16.11
<b>BC1</b>	-432.54	0.00	0.58	0.58	-482.67	0.00	0.61	0.61
<b>BC2</b>	-437.31	0.53	1.94	0.81	-478.90	0.68	2.08	0.72
<b>BC3</b>	-389.89	0.27	1.10	-0.11	-420.89	0.35	1.14	-0.16
<b>BC4</b>	-265.96	2.75	9.38	-3.09	-283.81	2.97	10.00	-3.49
<b>D1</b>	-5.19				3.71			
<b>D2</b>	3.18				-3.42			
<b>D3</b>	36.36				44.46			
<b>D4</b>	-44.94				-54.98			
<b>D5</b>	78.87				87.34			
<b>D6</b>	-111.93				-123.75			
<b>D7</b>	161.89				172.78			
<b>D8</b>	-205.59				-219.36			

As the table shows, both shear and bending moments are very similar in both cases since the whole structure behaves as a simply supported beam. However, with the axial forces there are quite different results, especially in the compression of the top chord, where the axial force in last part of it is near to zero in the axial restrained model (model 2). The explanation of it comes from the direction of the reaction force generated in the support, which should be almost parallel to the direction of the last brace (D8). So, it can be seen the higher values of axial on the braces of the second model due to the redistribution of the load between them.

It is also interesting to mention that the axial forces in the first two braces, brace 1 and 2 (D1 and D2 in Table 6.1), have different signs due to the value of the ‘reaction’ in the left part of the half truss (mid span) Anyway, those reactions are not going to affect to the truss behavior too much.

If the model 3 is compared with the simply supported structures (model 1 and 2) some bigger differences can be appreciate. Shear forces and moments continue being about the same, but there is even more changes in axial forces. The resultant in the top chord changes from a smaller compressive force in the beginning of it (TC1) and continues changing into a tensile effort in the last part (TC4). The bottom chord is also behaving quite different; it is also in tension in the first part (BC1, BC2 and BC3) but change into compression in the last part (BC 4 and BC5). It is due to the different way of distribute the load since it is a structure with the rotation restrained in both supports. It is almost totally fix structure, so the moment diagram of the whole structure is differ-

ent from the simply supported models (model 1 and 2). Those resultants are given in Table 6.2.

Table 6.2.- Stress resultants without fire using linear theory, model 3.

<b>MODEL 3</b>				
	$N$ [kN]	$Q$ [kN]	$M_{max}$ [kNm]	$M_{min}$ [kNm]
<b>TC1</b>	222.35	16.02	7.86	-7.74
<b>TC2</b>	197.84	16.03	8.21	-7.35
<b>TC3</b>	110.17	18.50	15.10	-5.51
<b>TC4</b>	-103.00	23.04	16.67	-15.47
<b>BC1</b>	-165.76	0.00	0.33	0.33
<b>BC2</b>	-155.97	0.61	1.52	0.30
<b>BC3</b>	-91.09	0.85	2.20	-0.91
<b>BC4</b>	52.81	1.70	4.79	-2.90
<b>BC5</b>	350.04	2.02	5.54	0.50
<b>D1</b>	10.04			
<b>D2</b>	-7.94			
<b>D3</b>	49.27			
<b>D4</b>	-61.78			
<b>D5</b>	93.18			
<b>D6</b>	-128.98			
<b>D7</b>	178.20			
<b>D8</b>	-231.89			

### 6.3 Roller model with fire consideration (model 1)

In this chapter a study and discussion about model 1 is done. It is, as mentioned in chapter 5, a simply supported structure without axial restraint. It is free to move laterally and thermal expansion consequences do not affect the structural response too much.

#### 6.3.1 Deflection at mid-span of the structure

In this point the deflection of the model 1 is discussed, especially in the mid-span (at the top chord) of the truss. Some different cases are going to be compared such as the 'linear' case ('*Linear*'), considering material non-linearities (*Nlmat*) and finally with full non-linearity of the structure (material and geometrical, *Nlmat&Nlgeom*). With '*linear*' it is referred to the situation where the elastic modulus change with temperature, but the material does not reach the yield. In *Nlmat* the material can reach the yield limit. This sort of fragmentation in the comparison will be used in all the different points of each model studied.

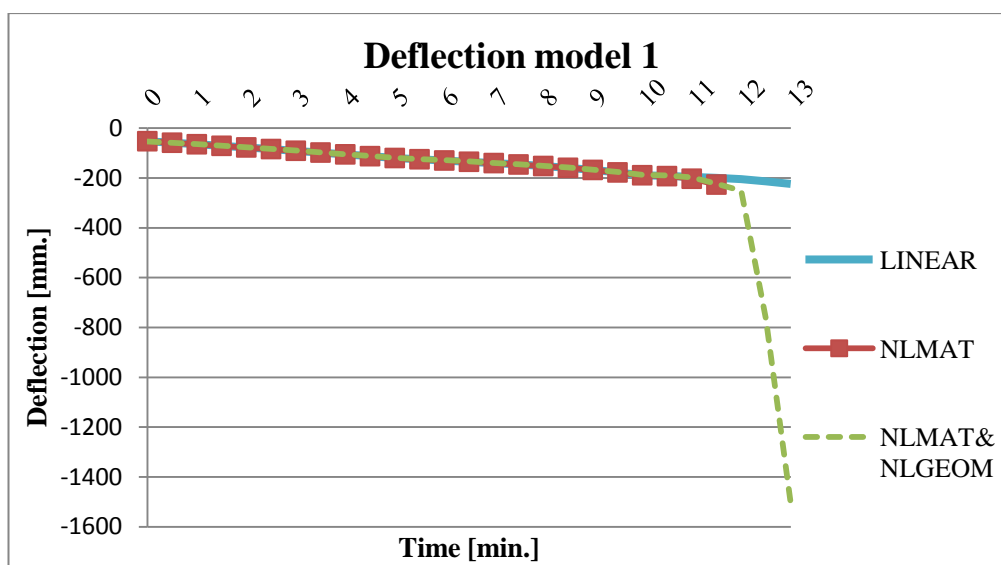


Figure 6.1.- Deflection over the time, model 1.

While the temperature grows, the elastic modulus start to decrease and steel resistances fall as well. It is the moment when the deflection starts to be important. As can be seen in Figure 6.1, when the geometrical non-linearities are considered, the model collapse after 13 minutes and when just material non-linearities are taken into account the model does not stand more than 11.5 minutes. The convergence stops in the *Nlmat* case because the yield in brace 6 and it is commented later. When the geometry non-linear behavior is considered the failure occurs because of the large displacement of the truss (about 1500 mm).

So comparing the different deflections in all the cases can be seen that the model behavior is almost the same until 11 minutes when the plastification of the brace 6 make the *Nlmat* case stop and in *Nlmat&Nlgeom* the deflection increases considerably to 1510 mm. It can be seen in Table 4.5 that the utilities of the braces 6 and 8 are 1.00 in the ambient conditions. The wall thickness of the brace 6 is 3 mm and the wall thickness of the brace 8 is 4 mm, so the failure at the brace 6 can be foreseen based on those values.

### 6.3.2 Axial forces of members

In this part the different axial forces achieved in the collapse moment are compared. For the '*Linear*' and *Nlmat&Nlgeom* the values are given after 13 minutes while the values for the *Nlmat* case are given after 11.5 minutes. In this model specifically, there are not going to be very big differences in axial forces until 11 minutes, but after this moment, the values of the forces in the braces should be different due to the plastification of the brace 6. It means that the '*Linear*' model could be quite accurate and reliable until this moment (11 minutes) after that more deep analysis would be needed to predict the behavior of the structure. The values of the axial forces of each case are shown in Table 6.3.

Table 6.3.- Axial forces at model 1.

	LINEAR	NLMAT	NLMAT& NLGEOM
	[kN]	[kN]	[kN]
<b>TC1</b>	435.58	435.99	431.75
<b>TC2</b>	419.49	418.19	394.15
<b>TC3</b>	352.27	350.14	299.56
<b>TC4</b>	165.66	168.49	145.56
<b>BC1</b>	-434.34	-434.64	-436.86
<b>BC2</b>	-434.14	-434.59	-427.94
<b>BC3</b>	-388.06	-383.21	-345.12
<b>BC4</b>	-266.31	-270.61	-238.38
<b>D1</b>	-0.16	-0.31	8.07
<b>D2</b>	-1.00	-0.86	-9.59
<b>D3</b>	35.87	39.46	74.88
<b>D4</b>	-43.32	-48.65	-70.60
<b>D5</b>	77.37	72.26	93.80
<b>D6</b>	-110.04	-101.33	-81.44
<b>D7</b>	162.00	164.18	148.18
<b>D8</b>	-205.94	-209.61	-179.07

The first two columns are about the same results, so it means that it was true that the linear case could be considered until 11 minutes. When the geometrical non-linearities are also considered, the axial forces are then quite different. The axial forces near the end of the truss (TC3, TC4, BC3 and BC4) are slightly lower and the forces of the diagonals are quite different. The forces of first four diagonals (braces 1, 2, 3 and 4) are larger and the last three (braces 6, 7 and 8) are quite smaller. As can be seen and compared using Table 6.4, there are some braces that are near to plastification.

Table 6.4.-  $N_{pl}$  of each member after 11.5 and 13 min.

	$T$	$N_{pl}$	$N_{pl}$
	[mm.]	(11.5 min.)	(13 min.)
<b>TC</b>	10	2275.21	1874.32
<b>BC</b>	8	1217.84	914.31
<b>D1</b>	3	64.57	48.69
<b>D2</b>	3	93.22	70.30
<b>D3</b>	4	267.14	195.62
<b>D4</b>	3	93.22	70.30
<b>D5</b>	5	489.51	345.98
<b>D6</b>	3	107.54	81.11
<b>D7</b>	5	489.51	345.98
<b>D8</b>	4	243.59	178.37

### 6.3.2.1 Imperfections considered in compressed braces

Since in this model, the structure is not restrained to lateral displacement, very high axial forces are not appearing due to the thermal expansions of the elements and, those members where the axial force was enough to make it collapse; they had tensile forces (brace 6). So, after running the ABAQUS analysis with some little bowl imperfections on the compressed braces (brace 3, 5 and 7), it displayed that the result were about the same than without this consideration. The imperfection due to the line load on the top chord was not big enough to cause buckling. So, it can be considered: no buckling problems will occur in this model and support conditions.

### 6.3.3 Bending moments of members

In the next tables (Table 6.5 and Table 6.6) are given the values of the bending moments on the truss near the collapse (11.5 for  $Nlmat$  and 13 minutes for ‘*Linear*’ and  $Nlmat&Nlgeom$ ). In these tables  $M_{max}$  means that the moment is the maximum of the member and negative means that the tension is under the center of the member.

Table 6.5.-Positive maximum bending moments for model 1.

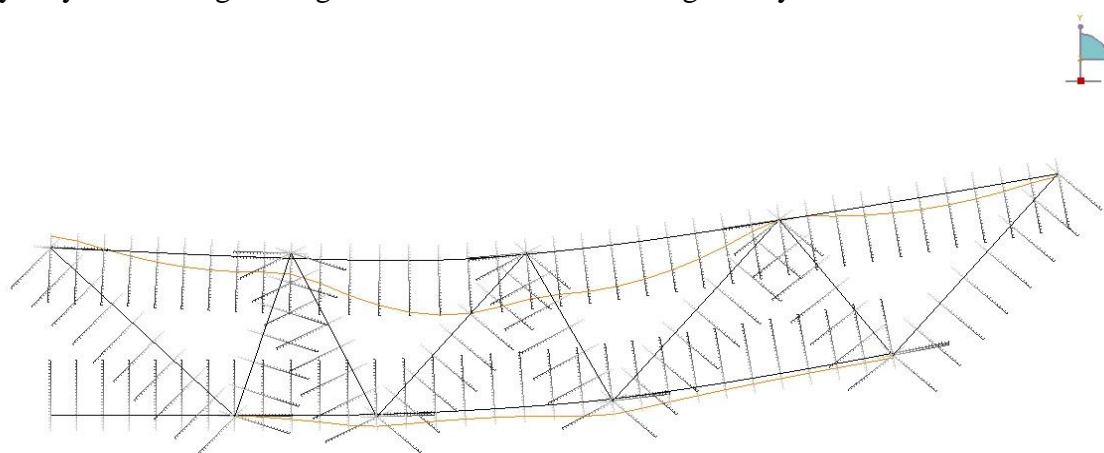
	$M_{max}$		
	LINEAR	NLMAT	NLMAT & NLGEOM
	[kNm]	[kNm]	[kNm]
<b>TC1</b>	12.84	13.96	18.34
<b>TC2</b>	-0.40	-1.05	-20.84
<b>TC3</b>	14.00	24.56	-3.11
<b>TC4</b>	15.56	26.77	3.72
<b>BC1</b>	0.88	1.07	0.57
<b>BC2</b>	3.26	3.39	13.72
<b>BC3</b>	1.71	3.90	25.94
<b>BC4</b>	9.43	9.72	22.71

As can be seen in Table 6.5, the moments change within the different analyses. The maximum moment at the top chord is reached with  $Nlmat$  and it is 26.77 kNm but it is smaller than the  $M_{pl}$  at this member and temperature (423 °C), which is 142.2 kNm. At the bottom chord is about the same, the maximum is achieved in the last part of the chord and it is 9.72 kNm (484.8 °C), much smaller than 59.06 kNm. The values in the  $Nlmat&Nlgeom$  analysis are larger, but they are also under the maximum value acceptable. With the minimum values is about the same. All of them get higher within the three columns but none of them are big enough to be worrying. Plastic moments after 13 min are 117.15 kNm (482.8 °C) for the top chord and 44.34 kNm (543.8 °C) for the bottom chord.

Table 6.6.-Negative minimum bending moments for model 1.

	$M_{min}$		
	LINEAR	NLMAT	NLMAT & NLGEOM
	[kNm]	[kNm]	[kNm]
<b>TC1</b>	-10.63	-9.30	-21.06
<b>TC2</b>	-14.73	-20.37	-83.95
<b>TC3</b>	-8.98	-12.43	-77.51
<b>TC4</b>	-15.88	-11.75	-23.63
<b>BC1</b>	0.88	1.07	0.16
<b>BC2</b>	1.15	1.32	1.52
<b>BC3</b>	1.55	1.60	10.41
<b>BC4</b>	-2.49	0.10	7.95

In Figure 6.2 are shown the bending moments at the *Nlmat&Nlgeom* case. It can be seen that the moments at the bottom chord are not very important and does not change a lot. However the moments at the top chord, especially between TC2 and TC3 are quite larger and more critical due to the redistribution of forces because of the plastification of the brace 6 and the large deflection (1500 mm. at that moment at the mid-span). Anyway they are not large enough to affect the structure dangerously.

Figure 6.2.-Bending moment diagram in *Nlmat&Nlgeom* after 13 min (ABAQUS).

## 6.4 Pin model with fire (model 2)

In this part model 2 is studied and discussed. As mentioned before, it is a simply supported structure with a hinge in the end of construction acting as a support. The lateral displacement is then restrained in that point, so it is not free to expand neither upon heating nor the action of other loads. The line load and especially the growing temperature will make the structure to elongate and since it was restrained against thermal expansion, the whole structure will probably start bowing down.

The behavior of this model will be quite more complicated than model 1 since some elements will experience opposite forces during the process. It will also be more sensi-

tive to the reduction of the elastic modulus and effects due to the axial restraint on the support.

#### 6.4.1 Deflection at mid-span of the structure

In this model, as commented above, is going to be very a large influence by the axial restriction combined with the steel thermal expansion. In Figure 6.3 can be seen the action of thermal expansion and how it changes the behavior of the whole structure, especially the deflection. The displacement without considering thermal expansion is about the same in both chords, while is the same magnitude, but opposite when the expansion of members is taken into account (see Figure 6.3).

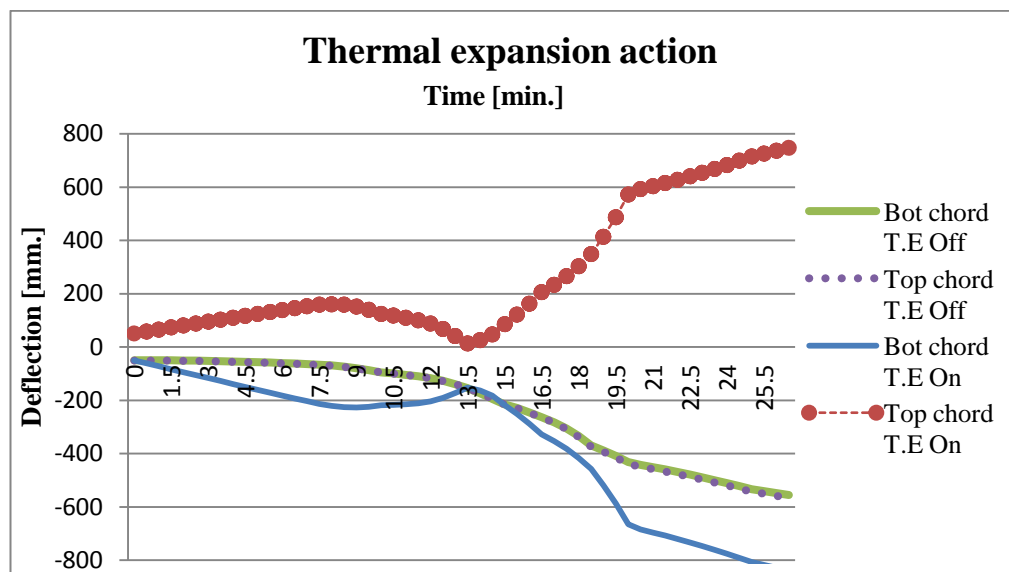


Figure 6.3.- Thermal expansion effects, model axially restrained. 'Linear' case.

Anyway, the bottom chord deflection before 10 minutes is almost the same in all the cases of study, so in the next figure (Figure 6.4) is shown the deflection experienced in the bottom chord while the temperature increases in all the different cases commented before ('Linear', *Nlmat* and *Nlmat&Nlgeom*). It can be seen how it changes after 10 minutes and that the convergence is reached in different time due to the geometrical non-linearity consideration, which makes a larger deflection possible.



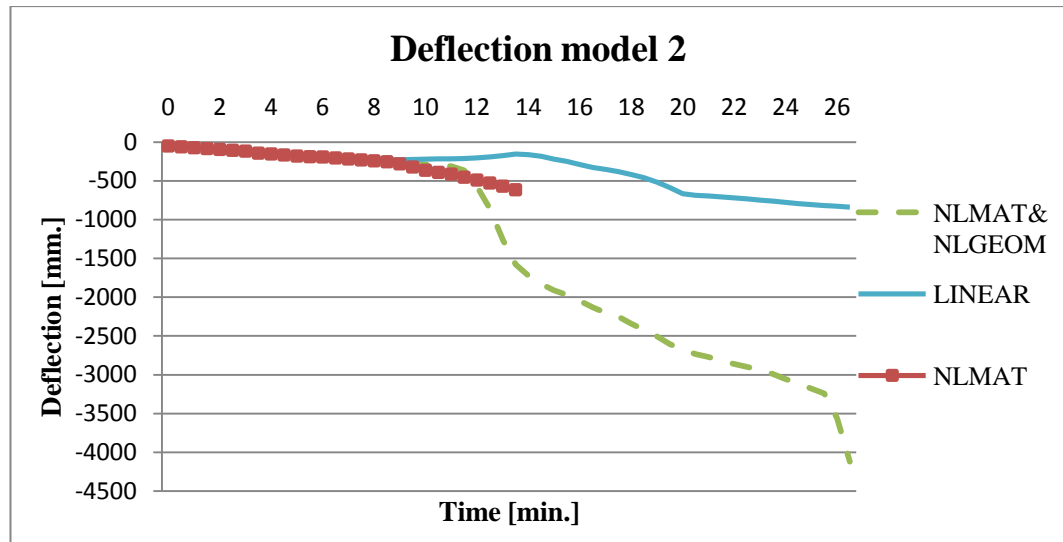


Figure 6.4.-Deflection over the time on axially restrained model, model 2.

The displacements *Nlmat&Nlgeom* are a little bit smaller if we compare it with *Nlmat* displacements before the 13.5 minutes (the collapse time for *Nlmat*). The *Nlmat&Nlgeom* convergence stop is achieved after 26.5 minutes with a maximum deflection of 4121 mm with the 'Linear' features the maximum deflection at 26.5 minutes is about 838 mm.

The axial forces are induced to the structure due to the thermal expansion. These forces will increase until a moment when the truss start to deflect considerably fast (around 12 minutes). Consequently, the thermal expansion will be very important in this model and while studying the axial forces and bending moments achieved while the different analyses, we could be able to understand how the model behave and how is it going to collapse. However, can be seen in Figure 6.4 that the same truss with full restraints at the ends can resist about twice the time in fire (26.5 minutes) compared to the truss without restraints (13 minutes).

#### 6.4.2 Axial forces of members

In this part the different axial forces achieved in the collapse are compared. Positive sign means compression and negative means tension. Axial forces are given at the point when the convergence stopped. In this case (Table 6.7), the variation between them could be not as reliable as before, due to the different times of collapse, since considering geometrical non-linearities, the model 2 would collapse after 26.5 minutes and after 13.5 with only material non-linear behavior. Anyway, both cases *Nlmat* and *Nlmat&Nlgeom* are compared with the 'Linear' case.

Table 6.7.-Stress resultants with all theories, model 2.

	LINEAR	NLMAT	LINEAR	NLMAT& NLEGOM
	[kN] (13.5 min.)	[kN] (13.5 min.)	[kN] (26.5 min.)	[kN] (26.5 min.)
<b>TC1</b>	4702.07	1338.83	1531.2	-136.38
<b>TC2</b>	4869.91	1364.96	1563.88	-148.27
<b>TC3</b>	5168.44	1383.85	1587.33	-177.42
<b>TC4</b>	5498.78	1303.17	1532.64	-236.40
<b>BC1</b>	1173.5	-91.57	-13.83	-135.70
<b>BC2</b>	931.85	-134.16	-89.54	-135.27
<b>BC3</b>	620.2	-179.12	-130.75	-116.19
<b>BC4</b>	320.48	-141.36	-166.76	-86.31
<b>D1</b>	-252.44	-44.94	-79.17	3.85
<b>D2</b>	185.99	31.75	58.04	8.94
<b>D3</b>	-237.37	-32.52	-31.37	25.69
<b>D4</b>	296.27	43.84	39.19	-11.64
<b>D5</b>	-193.58	22.64	8.09	27.00
<b>D6</b>	268.99	-34.97	-13.14	-23.04
<b>D7</b>	-196.97	85.73	70.79	54.15
<b>D8</b>	246.82	-109.52	-90.47	-60.43

It can be seen the big influence of the consideration of expansion, as said before. The forces reached now are quite larger than before. If the comparison between the ‘*Linear*’ and non-linear result is done, can be seen the important differences between theories. While with the ‘*Linear*’ analysis all the results are about the same at 8 and 13.5 minutes (Table 6.1), here the changes between model 1 and 2 are very significant. Also, if the resistances of the upper chord are checked with the hypothetical values reached in the ‘*Linear*’ case (where yield of members is not considered), 5498 kN is larger than 1874 kN (Table 6.4), so we can assume that the yielding starts at the upper chord at TC4 and make the *Nlmat* case collapse after 13.5 minutes. It is surprising that this part (TC4) near the support started with the axial load 1.83 kN at the beginning of the analysis.

It is also quite essential to study the possibility of buckling in this case of study (axially restrained) since some braces change from tension to compression in *Nlmat*, so they could be the cause of the beginning of the whole structure collapse. The buckling of member will be studied and discussed in the next part.

Looking at the last analysis, *Nlmat&Nlgeom*, it is shown that the structure (upper and lower chords) started all in compression and changes them all to tension before the collapse after 26.5 minutes with a maximum deflection of 4121 mm. It means that the catenary effect is acting in the structure, which collapse due to those big deflections.

### 6.4.2.1 Imperfections considered in some braces

A usual structure with no axial restrictions typical failure is due to the formation of plastic hinges that transform the structure into a mechanism. On the other hand, in axially restrained structures (externally non-statically determinate structures) will buckle some of its members under the axial forces before the material reaches the yield stress (Rotter & Usmani, 2000). In those members with thermal expansion considered and being axially restrained, like in this model, it takes place the development of high compressive forces which the beams cannot resist. Therefore, the investigation on whether or not buckling was the main cause was needed in this research. In that sense, dynamic analyses with imperfections have been performed in order to study this behavior and see how some of the braces would act while changing from tension to compression during the analysis.

In this part, a bow imperfection with the amplitude  $L/1000$  ( $L$  being the length of the member) has been introduced in some of the braces that seemed to buckle and those which change from tension to compression during the temperature growing analysis in order to magnify these buckling problems while the member is in compression. The braces that have been studied are brace 1, 2, 3, 4, 5 and 7. The axial forces of the chords also change from compression to tension, but the previous deformation due to the applied line load may act as an imperfection on them. So only the braces are previously deformed. The Euler formula (Equation 2.4) with reduction of the Young modulus will be used, and the critical buckling load was obtained with respect to time taking account the reduction of the properties of the members. Figure 6.5 illustrates the bottom chord deflection at the mid-span with and without bow imperfections.

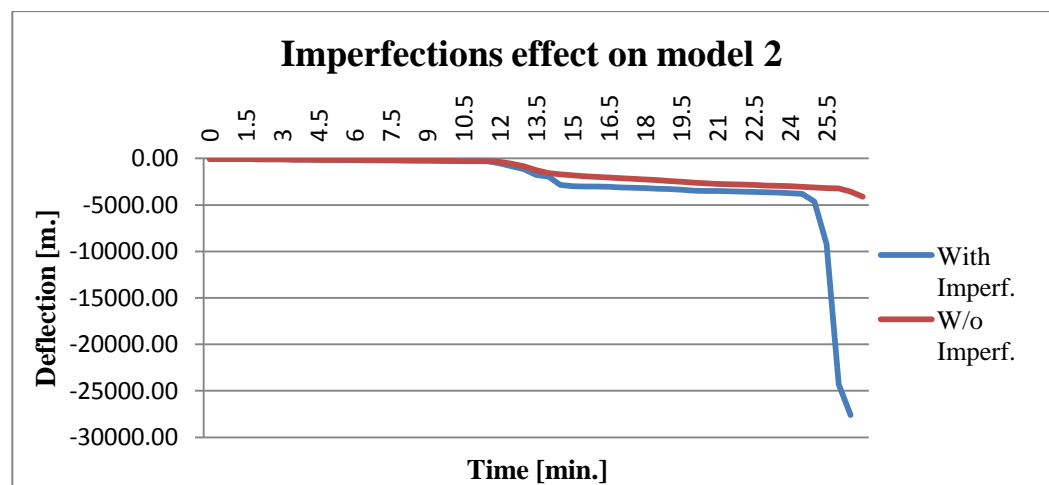


Figure 6.5. - Deflection at the mid-span with and without bow imperfections.

After analyzing the braces with the imperfection considered, neither of them had a plastic axial force resistance  $N_{bRd}$  smaller than the axial forces except the brace 7 which

after 20 minutes had a axial force around 14 % larger than  $N_{bRd}$ . The fact is that, this buckling has happened without the consideration of the imperfection. With the bow considered, the compression in brace 7 at this moment is lower. The rest of the braces, as commented above, are all with compressive forces under the  $N_{bRd}$ .

### 6.4.3 Bending moments of members

As can be seen in Table 6.8, bending moments are totally different between all the theories. The plastic moment for the upper chord after 13.5 minutes is 117.14 kN/m and it is a bit lower than the moment reached at the TC1 (121 kN/m), but the rest of them are all below the plastic moment.

Bending moments of the model 2 are given in Table 6.8.

Table 6.8.-Bending moments comparisson between theories, model 2.

	LINEAR		NLMAT		LINEAR		NLMAT& NLGEOM	
	[kNm] (13.5 min)		[kNm] (13.5 min)		[kNm] (26.5 min)		[kNm] (26.5 min)	
	$M_{max}$	$M_{min}$	$M_{max}$	$M_{min}$	$M_{max}$	$M_{min}$	$M_{max}$	$M_{min}$
<b>TC1</b>	121.01	-42.40	37.34	-31.96	16.93	-6.93	3.51	-12.40
<b>TC2</b>	8.99	-63.39	17.38	-35.09	4.84	-12.92	4.10	-7.95
<b>TC3</b>	4.47	-17.21	19.85	-1.13	10.87	-8.81	0.59	-6.86
<b>TC4</b>	-4.95	-28.57	10.53	-18.20	10.67	-18.14	2.26	-1.26
<b>BC1</b>	-3.26	-3.26	2.48	2.48	0.24	0.24	0.59	0.00
<b>BC2</b>	0.81	-3.48	5.35	3.52	1.59	0.79	0.88	-0.59
<b>BC3</b>	8.81	0.27	7.13	1.82	2.35	0.73	2.47	0.76
<b>BC4</b>	10.45	-10.95	5.09	0.20	4.21	0.28	2.83	1.04

Anyway, the moment diagram changes a lot if we compare all theories with maximum and minimum values. In the ‘Linear’ case are almost all moments larger than plastic moments except TC2 and TC3 at *Nlmat* (Table 6.8).

In conclusion, bending moments are very different between theories but are not as critical to the structure as the axial forces.

## 6.5 Fixed model with fire (model 3)

This structure is fully fixed at both ends. It is not free to rotate at its ends since it has a moment resisting support. Due to this, the structure will have a small deformation at the beginning of the heat action and the loading. Since it is not a statically determined structure, more deformation will be needed to reach the failure mechanism.

A great axial force and bending moment would be induced in members of this model due to the thermal expansion. These high stresses will make the structure reach the yield, then the axial forces will decrease slowly and deflection would start to get larger. The structure will be the most rigid of all the cases of study and the most difficult to get convergence with the program. The behavior of this structure is very sensitive to the reach of the yield stress of members; it would make it change very fast in that moment. This kind of structure should collapse far before the model 2 (only axially restrained) without as much deflection as the other (around 4 m).

### 6.5.1 Deflection at mid-span of the structure

In this structure, like in the model 2, the thermal expansion has a very important role in the deflection. It makes to the structure suffer very large axial forces which at the beginning are not enough to generate a big deformations, but quite different from the other two models. As can be seen in Figure 6.6, before 5 minutes the deflection is very similar with or without thermal expansion, but after this point it is completely different when the expansion of members is considered. Anyway the deflection in this model is not going to be as large as in the model 2 since it is a more restricted and supported model.

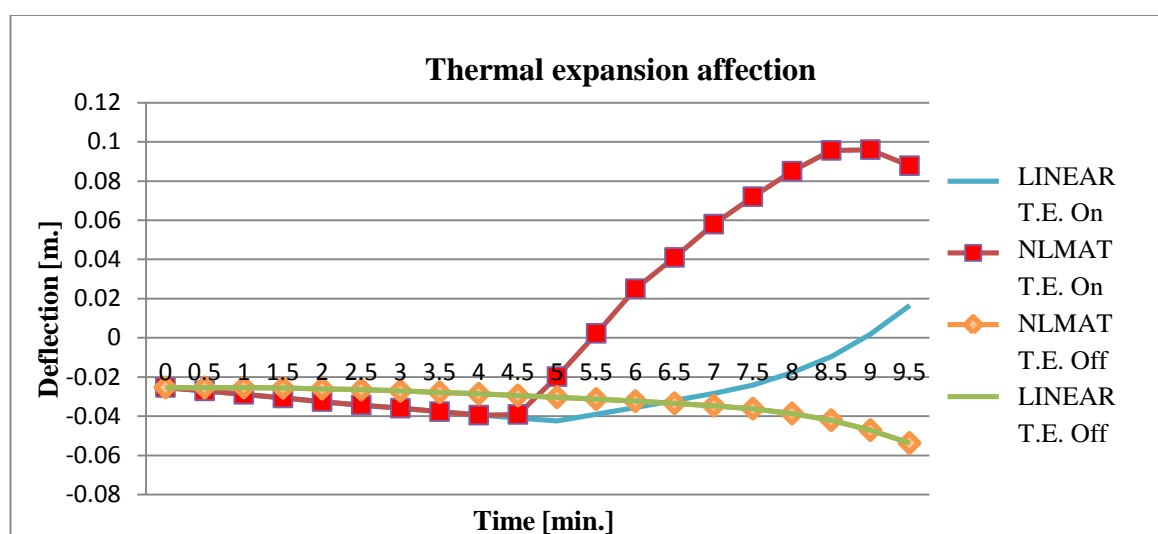


Figure 6.6.- Thermal expansion affection to model 3.

So, since the thermal expansion is real in structures and in our structure as well, the analyses have been done with this consideration. The real problem is the time that the structure could stand those studies in ABAQUS. It just ran until 9.5 minutes in *Nlmat* and only 3.5 with geometrical non-linearity. The deflection of the upper chord with respect to time is shown in Figure 6.7 and it shows very similar deflection until 3.5-5 minutes, but it starts to be quite different from there until the collapse of the *Nlmat* analysis (about a 70 % more of deflection in same time).

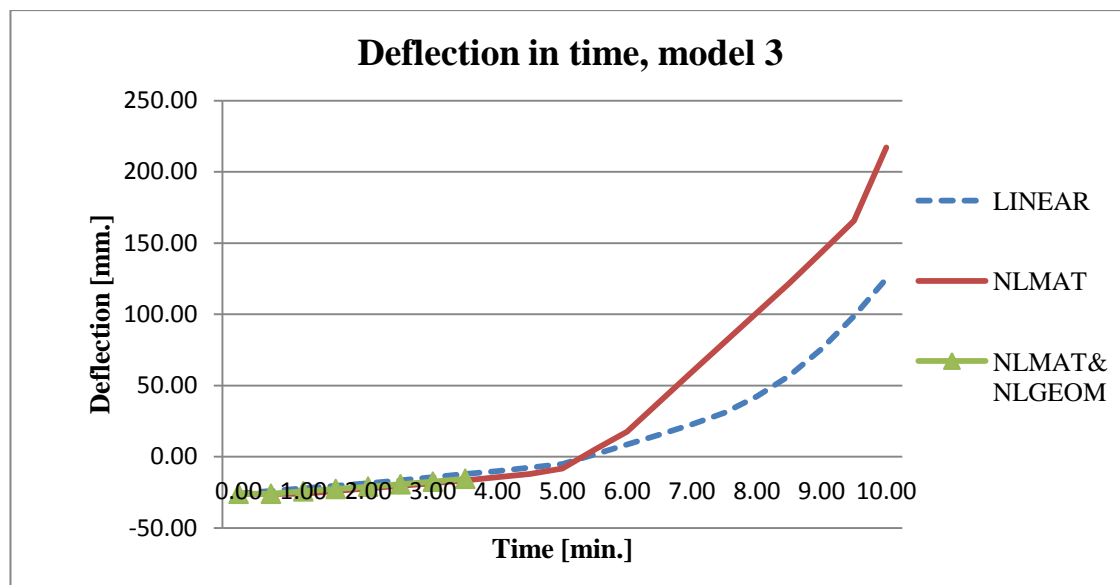


Figure 6.7. - Deflection in time, totally restrained model.

As can be seen, the structure hardly deflected at the beginning, around 20 mm until 4 minutes, when it started to deflect vertically and opposite to usual deflection, due to the influence of thermal expansion at the beginning. This thermal expansion of members creates the development of compressive axial forces, studied in the next part, and the different temperature also caused a change in bending moments.

### 6.5.2 Axial forces of members

In this part, where axial forces are studied, will be shown the variation of the forces in this model, especially due to the thermal expansion consideration. As can be seen in Table 6.9, axial forces are about the same after 3.5 minutes when the *Nlmat&Nlgeom* model stopped, and forces are larger than in the static linear analysis, where the maximum value of compressive axial force is 350 kN in the BC5, and after 3.5 min, it is about 1400 kN (about 400 % of the initial) at the upper chord.

Analyzing the model after 9.5 minutes is a bit different. It has already started to deflect vertically and positively due to the big compressive forces induced in the structure. As can be seen in the '*Linear*' analysis, TC4 reaches 4195 kN, much larger than 2292 kN, the maximum axial force that the upper chord can resist at this temperature. Also some of the braces (brace 1, 2, 4 and 6) have larger stresses than the  $N_{pl}$ . At this moment both chords are all at compressive situation, but at any moment it will start to change into tensile situation and deflection will grow fast.

Also, some of the braces have already started (between 3.5 and 9 minutes) to change from tension to compression, so they may have buckling problems.

Therefore, some of the members would be reach the yield, and there will probably become some plastic hinges in some elements. Anyway, this type of support would resist better than the others this hinge creation, and they may not cause the failure of the structure.

Table 6.9. - Axial stresses in the model 3 with different analyses.

	9.5 min.		3.5 min.	
	LINEAR	NLMAT	LINEAR	NLMAT& NLGEOM
	[kN]	[kN]	[kN]	[kN]
<b>TC1</b>	3660.90	1953.25	1441.10	1442.66
<b>TC2</b>	3786.11	2004.17	1469.26	1472.40
<b>TC3</b>	3998.85	2069.68	1488.73	1482.87
<b>TC4</b>	4195.49	2060.25	1420.91	1404.44
<b>BC1</b>	3254.65	1428.33	1090.93	1082.07
<b>BC2</b>	3061.99	1333.82	1028.16	1009.96
<b>BC3</b>	2837.87	1251.59	990.39	977.22
<b>BC4</b>	2640.21	1223.47	1012.26	1009.43
<b>BC5</b>	2467.65	1282.68	1143.80	1163.01
<b>B1</b>	-201.70	-99.08	-65.85	-74.99
<b>B2</b>	147.25	71.93	47.66	56.30
<b>B3</b>	-168.32	-61.05	-27.85	-26.58
<b>B4</b>	214.62	79.19	36.50	30.09
<b>B5</b>	-132.56	-21.87	12.30	18.51
<b>B6</b>	174.31	22.92	-20.77	-30.33
<b>B7</b>	-99.81	39.84	80.25	89.63
<b>B8</b>	137.48	-42.79	-101.54	-121.70

### 6.5.3 Bending moments of members

In this part, bending moments in the most restrained model are analyzed. Since in this model, rotation of both ends of the truss is not allowed bending moments are going to be higher than in other models. Anyway, as can be seen in Table 6.10 all the values are about the same except from the bending efforts in chords in ‘*Linear*’ and *Nlmat* after 9.5 minutes of fire acting, especially in the left part of the top chord (TC1 and TC2).

The values reached after 3.5 minutes are about the same (‘*Linear*’ a little bit higher in TC1 and TC2 but lower in TC3 and TC4 due to the different redistribution of models) in both analysis, ‘*Linear*’ and *Nlmat&Nlgeom* since there is not time enough to see important differences between the models.

Table 6.10. - Bending moment values of model 3 in different analyses.

LINEAR		NLMAT		LINEAR		NLMAT& NLGEOM	
[kNm] (9.5 min.)		[kNm] (9.5 min.)		[kNm] (3.5 min.)		[kNm] (3.5 min.)	
$M_{max}$	$M_{min}$	$M_{max}$	$M_{min}$	$M_{max}$	$M_{min}$	$M_{max}$	$M_{min}$
41.70	-22.66	29.42	-15.47	29.18	-7.31	27.56	-5.54
28.77	-37.03	6.20	-23.91	7.22	-15.19	6.49	-21.13
17.57	-4.10	14.90	-6.00	16.75	-4.29	21.63	-0.54
-21.88	-17.93	14.07	-16.52	16.55	-15.51	19.45	-26.13
0.00	-3.27	0.00	0.00	-0.45	-0.45	2.21	-4.61
0.42	-0.56	2.20	1.84	1.32	0.81	4.39	1.84
3.82	-6.00	5.25	-2.18	2.74	-1.01	2.34	-2.67
-0.84	1.11	10.27	-0.96	4.93	-1.78	6.15	-3.17
3.90	-10.70	-0.72	-7.93	-0.03	-0.29	2.98	0.52



## 7. CONCLUSIONS

Based on the results of the analyses commented above, some important ideas can be concluded about the behavior of structures with all the support boundary conditions studied using the different structural theories. In this thesis have been studied simply supported structures (model 1 and model 2) and moment resisting structures (model 3). The main question of this thesis was to see the differences between model 1 (externally statically determinate) and model 2 (externally non-statically determinate) especially and to see how their behavior is with linear and non-linear analyses.

Statically determined models (model 1) have not axial restraint, and therefore are not affected by the  $P-\Delta$  action as the other models. The behavior of this structure and its members is predictable and the collapse of the truss is mainly due to the plastification in some braces (brace 6 is the first), when they get the plastic maximum force, they just arrive to an unsustainable situation and the structure reaches failure.

On the other hand, non-statically determined models (model 2 & model 3) are axially restrained and are more influenced by the axial forces induced along the truss because of the thermal expansion of the members. A complex interaction between the bending moment, the deflection and the axial forces is happening and constantly changing during the fire exposure. The situations when the proportional limit is reached have a lot of significance in those models. They have this collapse procedure:

At the beginning all members are in high compression due to the thermal expansion consideration. After certain time of growing temperatures, the truss starts to bend since the elastic modulus starts to decrease and all the braces start to have buckling problems, starting in brace 6 (compression and not very thick brace). Then, the entire truss changes into tension due to the beginning of bending and the deflection become very large (4 metres in model 2) and the truss collapses because of a lot of deflection and large tensile forces. Fixed structures fail earlier than the pinned structure without as much deflection as the pinned truss, which reach very large deflections with a very important ductile behavior.

So, in statically determined trusses, the global behaviour is more predictable and the linear analysis fit very well with the non-linear analysis. So, in those kinds of structures, the linear theory could be used in the design. However, non-statically determined structures differs quite much from the linear behaviour and they have a behaviour which is

quite difficult to determine. Material and geometrical non-linear theories should be used when analysing these structures in fire. However, the statically determinate model (model 1) can be used to predict the same structure with restraints (model 2), but the result will be very conservative. In the case considered in this thesis model 1 resisted about 13 minutes and model 2 resisted about 26 minutes in the same fire with the same structure.

## REFERENCES

- Alpine Engineered Products, I. (2000). *Gecko Steel Truss, L.L.C.* Obtenido de <http://geckosteeltruss.net/Dictionary.htm>
- Boel, H. D. (2010). *Buckling Length Factors of Hollow Section Members in Lattice* .
- Buchanan, A. (2001). *Structural Design for Fire Safety* . University of Canterbury, Christchurch, New Zealand.
- Cedeno, G. A., Varma, A. H., & Gore, J. (2006). *Predicting the Standard Fire Behavior of Composite Steel Beams* . Indiana, USA.
- Chinnis, D. (2013). *Truss Destruction*. Colorado, USA.
- Choi, S., Burgess, I., & Plank, R. (2008). *Performance in fire of long-span composite truss systems. Engineering Structures*.
- Chung, T. T. (2010). *Analysis of Thermally Induced Forces in Steel Columns subjected to Fire*. Texas, USA.
- Dassault Systèmes Simulia Corp. (2009). *Getting Started with ABAQUS/CAE:Interactive Edition* .
- EN 1993-1-1. (2005). *Eurocode 3: Design of structures. Part 1-1 :General rules and rules for buildings* . Brussels, Belgium.
- EN 1993-1-2. (2005). *Eurocode 3: Design of structures. Part 1-2 :General – Structural fire design* . Brussels, Belgium.
- EN 1993-1-8. (2005). *Eurocode 3: Design of structures. Part 1-8 :General – Design of joints* . Brussels, Belgium.
- Flint, G., Usmani, A., Lamont, S., Torero, J., & Lane, B. (2006). *Effect of fire on composite long span truss floor systems. Journal of Constructional Steel Research*.
- Heinisuo, M., Möttönen, A., Paloniemi, T., & Nevalainen, P. (1999). *Automatic design of steel frames in a CAD-system, Proceedings of the 4th Finnish Mechanics days*. Lappenranta, Finland.
- Heinisuo, M., Tiainen, T., & Jokinen, T. (2013). *Tubular truss design using steel grades S355 and S420* . Tampere University of Technology, Tampere, Finland.

- Jalkanen, J. (2007). *Tubular Truss Optimization Using Heuristic Algorithms* . Tampere University of Technology, Tampere, Finland.
- Kellert, S. H. (1993). *In the Wake of Chaos: Unpredictable Order in Dynamical Systems*. . University of Chicago, Chicago, USA.
- Kukkonen, J., Heinisuo, M., & Toumala, M. (2005). *Structural calculations of cold-formed steel structures in an integrated system* . Master of Science Thesis. Department of Built Environment. Tampere University of Technology, Tampere, Finland.
- Liu, M., Zhao, J., & Jin, M. (2010). *An experimental study of the mechanical behavior of steel planar tubular truss in a fire*. *Journal of Constructional Steel Research* . China.
- Martini, K. (2010). *Introduction to Structural Design* . Virginia, USA.
- Mela, K., Heinisuo, M., & Tiainen, T. (2012). RUOSTE. *Weight and cost optimization of high strength steel tubular trusses* . Tampere, Finland.
- Narang, V. A. (2005). *Heat Transfer Analysis In Steel Structures* . Worcester, USA.
- Ongelin, P., & Valkonen, I. (2012). *Rakenneputket. EN 1993-Käsikirja, Ruukki*.
- Pada, D. (2012). *Steel skeleton behaviour in decaying fire*. . Licenciate thesis. Tampere University of Technology, Tampere.
- Quintiere, J., di Marzo, M., & Becker, R. (2002). *A suggested cause of the fire-induced co-lapse of the World Trade Towers*, *Fire Safety Journal*. Fire Protection Engineering Department, University of Maryland, College Park, MD, USA.
- Rotter, J., & Usmani, A. (2000). *Fundamental Principles of Structural Behavior Under Thermal Effects*. *Fist International Workshop on Structures in Fire* . Copenhagen.
- Ruukki. (2012). *Rakenneputket*. Rautaruukki Oyj.
- Seputro, J. (2006). *Effect of support conditions on steel* . Research Project Report. Department of Civil Engineering. University of Canterbury, Christchurch, New Zealand.
- Simulia, D. S. (2011). *ABAQUS analysis user's manual* . Providence, RI, USA.
- Timoshenko, S. P. (1953). *History of Strength of Materials*. New York, USA., New York, USA.: McGraw-Hill Book Company Inc.

Timoshenko, S., & Goodier, J. (1951). *Theory of Elasticity*. New York, Toronto, London: McGraw-Hill Book Company Inc.

Usmani, A., Chung, Y., & Torero, J. (2003). *How did the WTC towers collapse: a new theory*. *Fire Safety Journal*. Fire Safety Journal, Volume 38, Issue 6.

Yang, Y., Lin, T., Leu, L., & Huang, C. (2008). *Inelastic postbuckling response of steel trusses under thermal loadings*. *Journal of Constructional Steel Research*.